Production planning and Order Acceptance: an Integrated Model with Flexible Due Dates

Nadjib BRAHIMI
Département d'Automatique, Productique et Informatique
Ecole des Mines de Nantes
Nantes, France
Nadjib.brahimi@mines-nantes.fr

Tarik Aouam
Department of Business Informatics and Operations Management, Faculty of Economics and Business Administration, Ghent University
Ghent, Belgium
Tarik.Aouam@ugent.be

El-Houssaine Aghezzaf
Department of Industrial Management, Faculty of Engineering and Architure,
Ghen University, Gent, Belgium
ElHoussaine.Aghezzaf@ugent.be

***Abstract*— We study a tactical problem integrating production planning with order acceptance decisions. We explicitly consider the dependency between the workload (and work-in-process inventory) and lead times. In the new model, orders are accepted/rejected and their processing period is determined. This problem is formulated as a mixed integer linear program for which two relax-and-fix heuristic solution methods are proposed. The first one decomposes the problem based on time periods while the second decomposes it based on orders. The performances of these heuristics are compared with the performance of a commercial solver. The numerical results show that the time-based relax-and-fix heuristic outperforms the order-based relax-and-fix heuristic and the solver solution as it yields better integrality gaps for much less CPU time.**

Keywords*— *Production Planning; Order Acceptance; Clearing Functions; Load-Dependent Lead Times, Flexible Lead Times; Relax-and-Fix, Delivery Time Windows

I. INTRODUCTION

The primary objective of classical production planning models is to satisfy customer demands while minimizing production costs or maximizing profit. Usually, orders are grouped (aggregated) at the tactical level to simplify the decision making process. However, it is often important to distinguish customer orders for several reasons (See [1] and [2]). Firstly, even if the finished good is the same, different customers might impose particular conditions on the source of raw materials or on the quality control tests to be carried out during the manufacturing process of their orders. Secondly, in the case of limited capacity, the production planner can only satisfy demands partially and consequently has to decide which orders to satisfy.

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Even when there is enough capacity it is not necessarily interesting to accept orders. Indeed, there are two fundamental assumptions in traditional production planning models: (*i*) the production lead times are constant and do not depend on the workload, (*ii*) and in any given period, the shadow price of the capacity constraint is equal to zero when there is enough capacity (capacity constraint is not binding); this means that the cost of adding one unit (or order) to the production stage is zero as long as the capacity limit is not reached. As a consequence of these assumptions, production planning models try to satisfy as many customer orders with known due dates as production capacity permits.

Actually, production lead-times depend on the workload. Queuing models have revealed that lead-time increases non-linearly as the resource utilization approaches 100% [3] [4]. Therefore, the more orders are accepted the higher are the production lead times, resulting in the possibility of missing customer due dates. This means that the planner can be faced with situations where production capacity is available but the next orders should be rejected in order not to delay some already accepted customer orders.

In addition, even if the unit price the customer is willing to pay exceeds the variable production cost and there is enough capacity to avoid shortage, the decision whether a customer order should be accepted or not is not always straightforward. There are two possible arguments to support this fact. The first argument has to do with economies of scale. In fact, in the case of high fixed or set-up costs it might not be economical to satisfy a single order of a small quantity. The order must be aggregated with additional orders to justify the production setup [5]. The second argument has to do with the workload of the production stage. Kefili et al. [6] show that the marginal prices of capacitated resources are not necessarily equal to zero when the utilization is less than one. This means that even in the case where capacity is available, the revenue from an additional order should at least offset the variable production cost plus the shadow prices of the capacity constraints that take into account workload.

Based on these arguments, we can conclude that models that integrate production planning decisions with load dependent lead times and order acceptance decisions have a great potential to improve the overall profitability of the firm. Furthermore, by allowing due date flexibility, more orders can be accepted resulting in higher profits and more reliable delivery dates (lower delays). In this research work, we integrate order acceptance and production planning decisions in a single model, while considering flexible due dates and load dependent lead times. The considered problem is formulated as a mixed integer linear program (MILP). When the number of orders and the number of periods increase, and for certain parameter settings it becomes difficult if not impossible to obtain good solutions in reasonable computation times. We propose relax-and-fix heuristics to solve efficiently large instances of the problem.

The remainder of the paper is organized as follows. A literature review is presented in Section II. In section III, the production planning problem with order acceptance decisions and flexible due dates is presented. In section IV, two relax-and-fix heuristics are presented. Section V presents some numerical experiments to evaluate the proposed heuristics. Some concluding remarks are presented in Section VI.

II. Literature Review

The dependency between resource utilization and lead times (or equivalently available capacity) has already been addressed to some degree by some authors. Voss and Woodruff [7] propose a nonlinear model where the function linking lead time to workload is approximated by a piecewise linear function. use Clearing functions (CFs) were used by [8], [9], [10], and [11] to model the dependency between workload and lead times. Several related models are proposed in the recent book by [12]. Production planning models with load-dependent lead times are reviewed by [2] and [13]. Aouam and Uzsoy [14] compare the performance of various production planning models with workload-dependent lead times under demand uncertainty. In this paper, a CF is used to model the capacity of the production stage in order to relate the production workload resulting from all accepted orders to the production lead-times.

Linear programming based production planning models typically consider fixed lead times or time lags and represent capacity as a fixed upper bound on the number of hours available at the resource in a period [7]. However, these lead times or time lags are independent of workload. As an alternative, load-dependent production planning models with clearing functions (CFs) capture the relationship between workload and output at a capacitated production resource [9] [10] [15]. A CF represents the relationship between the average workload of a production resource, usually some measure of work in process inventory (WIP), and the average throughput of the resource in a planning period. For most capacitated production resources subject to congestion, limited capacity leads to a CF that is concave and increasing [13]. The CF, denoted by $f(.)$ that is increasing and concave with $f(0) = 0$, relates the throughput to the WIP as follows,

$$X_t = f(\overline{W}_t) \qquad \forall t \qquad (1)$$

where $\overline{W}_t = W_{t-1} + R_t$ represents the resource load for period t, or the total amount of work that becomes available for processing during the period. Following [8] and [11], and for tractability reasons, the CF is approximated using an outer linearization. In fact, $f(.)$ can be approximated by the convex hull of a set of affine functions of the form,

$$\hat{f}(W) = \min_{k=1\dots K}\{a_k W + b_k\} \qquad (2)$$

a_k and b_k are the slope and intercept of segments $k \in \{1 \dots K\}$

Ivanescu et al. [16] consider the order acceptance problem in the batch industries where the processing times are uncertain. The authors use regression based models in order to determine whether there is enough capacity to accept a customer order with the due date requested by the customer. Geunes et al. [17] consider a production planning problem with order acceptance and call it the order selection problem. The uncapacitated case is solved using a polynomial time algorithm and they propose a Lagrangian relaxation approach for the capacitated case. For a more extensive review of order acceptance literature the reader is referred to [18]. Aouam and Brahimi [1] present a robust model that integrates production planning with load dependent lead-times and order acceptance decisions, which considers demand uncertainty and where a fraction of the order quantity can be accepted.

The subject of lead time or due date flexibility is directly related to demand time windows. The latter are grace periods (allowed by the customers) during which the order can be delivered without penalty. To the best of our knowledge, the first production planning models with demand time windows were introduced by [19]. They proposed dynamic programming algorithms to solve uncapacitated lot sizing problems with and without backlogging. Charnsirisakskul et al. [20] propose an order acceptance model where they show the economic benefits of lead time flexibility. They solve a capacitated example using the commercial solver CPLEX. Merzifonluoğlu and Geunes [21] propose a similar model with production setup decisions. The uncapacitated case is solved using a dynamic programming algorithm, while the authors propose heuristics to solve the general case. This stream of work emphasizes the integration of order acceptance decisions in production planning decisions to take into account economies of scale achieved per setup when orders are aggregated. Recently, Brahimi [22] considered the issue of integrating order acceptance decisions with due date flexibility. He presents two heuristic solutions for the problem: a reversals heuristic and a relax-and-fix heuristic based on order decomposition. The present paper improves these heuristics and presents a new time based relax and fix heuristic that outperforms them in terms of integrality gap and CPU times. The current work and [22] were combined, extended, and submitted to an international journal [23].

Relax-and-fix heuristics were applied to different production planning problems including the capacitated single level multi-item lot sizing problem [24], the multi-level lot sizing problem [25], and the lot sizing and scheduling problem with parallel machines [26]. Most implementations of relax-and-fix heuristics in production planning consider partitioning the time horizon and forward or backward fixing integer variables (ex. [25] and [27]).

Compared to previous work, our model considers more realistic capacity constraints that reflect the dependency between workload, affected by the number of accepted orders, and production lead times. The models also incorporate flexible due dates that allow production smoothing, increase the number of accepted orders, and determine reliable due dates. Furthermore, two relax and fix heuristics are proposed and compared: one decomposes the problem based on time periods and the other based on customer orders. The latter heuristic incorporates reversals, which are inspired by the sub-tour reversals heuristic for the traveling salesman problem [28].

III. Production Planning Model with Order Acceptance, Load DePendent Lead Time and Flexible Due Dates

The load-dependent production planning model determines production decisions to satisfy customer orders and maximize the total profit. Each order i is characterized by an order size q_i, reservation price or marginal revenue π_i, and a lost sales cost l_i. We allow due date flexibility in our model, i.e., the due date required by the customer is given as a set of possible dates rather than a fixed date, which leads to a win-win situation for the firm and customers. In fact, this flexibility when captured in production planning models results in more accepted orders, smoother production plans, higher profits, and more reliable due dates (lower delays). In this setting, a customer provides a time window with earliest delivery date e_i and a latest delivery date f_i.

For a given order i, the customer provides a time window with earliest delivery date e_i and a latest delivery date f_i. If the order is accepted it can only be delivered in period $t \in [e_i, f_i]$.

The decision variables in the model are, for each period: the quantity released R_t, the production level X_t, the Work-In-Process (WIP) level W_t, and the inventory level I_t. The marginal costs are: release cost r_t, processing cost c_t, WIP holding cost w_t, inventory holding cost h_t, and backlogging cost p_t. Also, let the binary variable S_{it} such that $S_{it} = 1$ if order i is accepted and to be satisfied in period $t \in [e_i, f_i]$ and $S_{it} = 0$ otherwise. The integrated production planning and order acceptance model with flexible due dates can be formulated as follows:

PP-OA-FDD

Objective function:

$$Maximize\ P^{FDD}(R_t, X_t, W_t, I_t) = \sum_i \pi_i q_i \sum_{t=e_i}^{f_i} S_{it} - \sum_t (r_t R_t + c_t X_t + w_t W_t + h_t I_t) - \sum_i q_i l_i \left(1 - \sum_{t=e_i}^{f_i} S_{it}\right) \quad (3)$$

Subject to constraints:

$$W_t = W_{t-1} + R_t - X_t\ , \quad t = 1, \dots, T \quad (4)$$

$$I_t = I_{t-1} + X_t - \sum_i q_i S_{it} \quad t = 1, \dots, T \quad (5)$$

$$X_t \leq a_k (W_{t-1} + R_t) + b_k \quad t = 1, \dots, T \ \wedge \ k = 1, \dots, K \quad (6)$$

$$\sum_{t=e_i}^{f_i} S_{it} \leq 1 \qquad i = 1, \dots, N \quad (7)$$

$$R_t, X_t, W_t, I_t, \geq 0 t = 1, \dots, T \quad (8)$$

$$S_{it}: binary \qquad t = 1, \dots, T \ \wedge \ i = 1, \dots, N \quad (9)$$

The objective function in equation (3) maximizes the total profit P^{FDD} over the planning horizon. Constraints (4) and (5) define WIP and finished goods inventory balances, respectively for each period. Constraints (6) represent the capacity constraints defined by the CF. Constraints (7) ensure that order i can only be accepted and satisfied within the customer specified time window $[e_i, f_i]$. Non-negativity and integrality constraints are defined by (8) and (9).

Solving this model using state of the art MILP solvers is not efficient for large size problems. Thus, we propose a MILP-based heuristic to solve it efficiently.

IV. Heuristics for Solving PP-OA-FDD

A. *General structure of the heuristics*

For problems of realistic sizes, with a large number of planning periods and orders, problem PP-OA-FDD is very hard to solve in reasonable computational times. This section presents two relax-and-fix heuristics to tackle this difficulty. The first heuristic decomposes the problem based on time periods while the second decomposes the problem based on customer orders. In relax-and-fix heuristics, the integer variables in a MILP formulation are separated into subsets. The heuristic usually proceeds by fixing a subset of variables, usually the most important ones, and relaxing the integrality of the other variables. Then, it gradually fixes the relaxed variables [29] [30]. The only integer/binary variables in PP-OA-FDD formulation are S_{it} variables and thus the problem can be decomposed over orders ($i = 1..N$) or over time periods ($t = 1..T$) or over both of them.

B. *Time-based relax-and-fix heuristic*

In the time based decomposition, integrality constraints are imposed on variables S_{it} ($\forall\ i = 1..N$) within a *decision time interval*, which is an internally rolling horizon. In any iteration of the relax-and-fix heuristic, the time horizon is partitioned into three intervals: a decision time window, a frozen interval preceding the decision time window and consisting of periods with variables that are fixed, and an approximation interval after the decision window where the binary constraints are relaxed. The two main parameters of this approach are: the size of the decision time interval (α) and the size of the frozen interval ($\beta \leq \alpha$). In any iteration, the decisions of the first β periods in the current decision window will be frozen in the following iteration. Figure 1 illustrates the three sub-intervals for $\alpha = 5$ and $\beta = 3$. In each iteration, an optimal or heuristic solution is obtained using a MILP solver. Two stopping parameters need to be determined for the heuristic: the minimum integrality gap and the maximum allowed CPU time in each iteration.

C. Order-based relax-and-fix heuristic

The main difference between the order-based decomposition and the time-based decomposition is that intervals are naturally identified in the latter because of the chronological nature of time periods, while a sequence of orders ($I = \{i_1, i_2, \dots, i_N\}$) needs to be determined. Then the relax-and-fix heuristic is applied on a given sequence and results in a given feasible solution of the problem instance. Several sequences of orders are constructed and evaluated.

An initial sequence is obtained using a Most Profitable First (MPF) priority rule. In the MPF rule, initially, all orders are supposed to be released and satisfied on their earliest due date, which yields a unit profit of $\pi_i^{'} = \pi_i - r(\tau_i)$ for each order i and the sequence ($I = \{i_1, i_2, \dots, i_N\}$) is obtained by sorting the orders in decreasing order of unit profit $\pi_i^{'}$ using QuickSort function as shown in Algorithm 1. For this sequence, the relax-and-fix heuristic is applied in such a way that the decision subset corresponds to the first $\alpha^{'}$ orders. The frozen subset is $\beta^{'} \leq \alpha^{'}$ and the decisions corresponding to the rest of the orders belong to the relaxed subset.

After updating the best solution, other sequences are constructed using the reversals heuristic, subroutine *Reverse*. When the initial sequence is reversed two-by-two, the resulting new sequences are: $I = \{i_2, i_1, \dots, i_N\}$, $I = \{i_1, i_3, i_2, \dots, i_N\}$, …, $I = \{i_1, i_2, \dots, i_N, i_{N-1}\}$. The best reversal and solution value are saved. The best sequence in the two-by-two reversal is used as a starting point for a three-by-three reversal. Supposing that the best solutions obtained for sequence $\{i_1, i_3, i_2, i_4, i_5, \dots, i_N\}$ in the two-by-two reversals, in the three-by-three reversals, the generated sequences are $\{i_2, i_3, i_1, i_4, i_5, \dots, i_N\}$, $\{i_1, i_4, i_2, i_3, i_5, \dots, i_N\}$, $\{i_1, i_3, i_5, i_4, i_2, \dots, i_N\}$,.., and $\{i_1, i_3, i_2, i_4, i_5, \dots, i_N, i_{N-1}, i_{N-2}\}$. The best sequence in the three-by-three reversals is the starting point of a four-by-four reversals and so on.

Algorithm 1: RerversalsHeuristic

```
BestSequence ← QuickSort(Orders);
BestProfit ← −∞
for Reversals ← 1 until N do
    if Reversals = 1 thenMaxReversals ← 1
    else     MaxReversals ← N-Reversals+1
    end-if
    ReversalBestProfit ← −∞
    for ReversePoint ← 1 until MaxReversals do
        for i ← 1 until N do S[i] ← 0;
        flag ← true;
        if (Reversals > 1) then
            Reverse(BestSequence,ReversePoint,
            ReversePoint+Reversals-1)
        end-if
        SequenceBestProfit ← −∞
        Relax-and-fix(α′ , β′, sequence)
        if (SequenceBestProfit ≥ ReversalBestProfit) then
            ReversalBestProfit ← SequenceBestProfit;
            UpdateBestSequence();
        end-if
    end-for
    if (ReversalBestProfit ≥ BestProfit) then
        BestProfit ← ReversalBestProfit;
        UpdateBestSequence();
    end-if
end-for
```

The Relax-and-fix($\alpha^{'}, \beta^{'}$) subroutine (Algorithm 2) forces the integrality condition on binary variables of the first $\alpha^{'}$ orders with the highest profit $\pi^{'}$ and relaxes the other binary variables. Then, it permanently fixes the solution for the first $\beta^{'}$ variables ($\beta^{'} \leq \alpha^{'}$), sets integrality constraints on variables indexed from $\beta^{'} + 1$ to $\beta^{'} + \alpha^{'}$ and relaxes integrality for orders after $\beta^{'} + \alpha^{'} + 1$. The process is repeated until the last order in the sorted list is reached. Furthermore, compared to simple relax-and-fix heuristics, our heuristic applies the reversals function and explores more possible solutions. The heuristic's main inputs are the number of orders for which the integrality constraints are to be respected in each iteration ($\alpha^{'}$) and the number of orders for which the binary variables are to be permanently fixed in each iteration ($\beta^{'}$). The first step of the heuristic calculates the number of

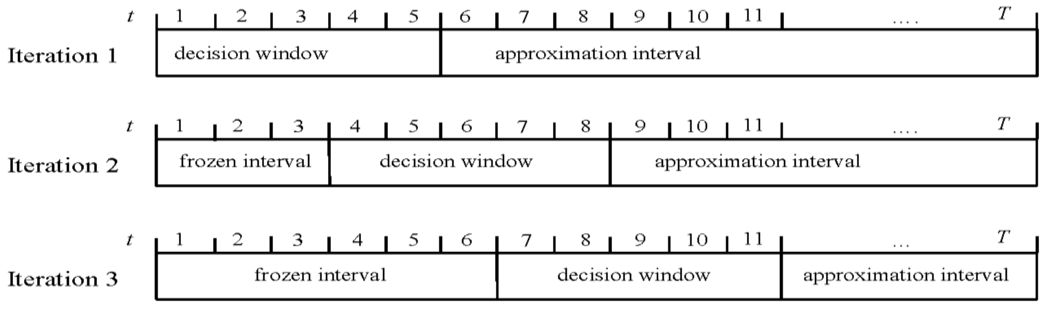

Figure 1. The different intervals in a time-based decomposition of a relax-and-fix heuristic

iterations based on these two parameters. Then, starting from the beginning of the sequence of the sorted orders, the sub-problems are solved until all binary decision variables are fixed.

Algorithm 2: Subroutine Relax-and-Fix(α' , β', Sequence)
Input: α', β'
Caculate NumIter
for i ← 1 **until** NumIter
Relax binary variables of orders after the last α' interval
Solve the sub-problem
Permanently fix S_{it} variables for orders within β'
end-for;

V. EXPERIMENTAL RESULTS

This section evaluates the efficiency of the proposed heuristics. The optimization models as well as the heuristics have been implemented in Xpress-IVE version 1.24 and run on a PC with intel CORE i7-2.4Ghz microprocessor and 16GB RAM.

A. Generated data sets

Data sets were generated with N =20 to 500 orders received for a period of T =10 to 100 periods. The production related unit costs are r_t =\$3, while c_t =0, w_t =\$35, and h_t =\$15, $\forall t$. The unit profit is equal to 100, 110 and 115 for small, medium, and large size orders, respectively. The earliest delivery date of each order is generated from a uniform distribution between 1 and T. The size of each order is generated from a uniform distribution between $\frac{1}{2}\bar{q}$ and $\frac{3}{2}\bar{q}$, where:

$$\bar{q} = \frac{T \times b_K \times DC}{N}$$

DC is the total orders over the nominal capacity for the whole planning horizon, i.e. $DC = \frac{\sum_i q_i}{T \times b_K}$.

The lost sale cost per unit is: $l_i = 1.2 \times \pi_i$. The intercepts and the slopes of the clearing function are defined as $(a_k, b_k) = (0.5, 0), (0.069, 136), (0.036, 154.8), (0, 180)$ for $k = 1, \ldots, 4$. The analysis of the performance of the heuristics was based mainly on capacity tightness determined by coefficient $DC = \frac{\sum_i q_i}{T \times b_K}$ and order time window $\Delta_i = f_i - e_i + 1$. DC was varied between 0.6 (loose capacity) and 1.2 (demand exceeding capacity). To analyze the impact of due date flexibility, Δ_i was set to $\Delta_i \in \{2, 4, 6\}$.

B. Analysis of the performance of the heuristics

1) Preliminary tests

The PP-OA-FDD model was solved using the time-based relax-and-fix heuristic and using order-based relax-and-fix heuristic (Algorithm 1). The parameters used for each heuristic are summarized in Table 1. The numerical experiments were carried out on both small size and large size instances. The stopping criterion used in each iteration of the two heuristics is the minimum integrality gap, which is set to 0.1%.

Preliminary tests were carried out on small size problems. In these problems there are $N = 20$ orders to be scheduled over a planning horizon of $T = 10$ periods. Order time windows were set to $\Delta_i \in \{2, 4, 6\}$ and capacity tightness was set to $DC \in \{0.6, 0.9, 1.2\}$. For each setting (given values of Δ_i and DC), five instances were randomly generated. The performance of the heuristics is measured using the gap between the optimal solution (Opt) obtained using the solver and the heuristic solution (Sol):

$$Gap = 100 \times \frac{Opt - Sol}{Opt}$$

Table 2 shows the gaps obtained by the heuristics. The CPU times are shown on the last raw of the table.

Table 1. Parameters of the heuristics

	Order-based heuristic		Time-based heuristic	
	RF-N-10-8	RF-N-15-10	RF-T-5-3	RF-T-10-8
α	10	15	5	10
β	8	10	3	8
Min integrality gap	0.1%	0.1%	0.1%	0.1%

The RF-T-10-8 heuristic outperforms all other heuristics in terms of quality of solutions. In fact, for some parameter settings it is able to find the optimal solutions for all the generated instances. However, it requires the largest CPU time on average when compared to other heuristics. Heuristic RF-T-5-3 might be considered as a good compromise between CPU time and solution quality. The solver on the other hand requires an average CPU time of 6.25 seconds and a maximum CPU time of 900 Seconds (maximum allowed execution time) to find the optimum, while RF-T-10-8 heuristic requires an average time of 2.57 Seconds and a maximum time of 97 Seconds to reach an average gap of 0.01 %.

Table 2. Gaps and CPU times for small size problems

	Para-meter	Value	RF-N-10-8	RF-N-15-10	RF-T-5-3	RF-T-10-8
Gap (%)	Δ_i	2	1.09	0.36	0.46	**0.00**
		4	1.80	0.40	0.16	**0.01**
		6	2.02	0.39	0.19	**0.01**
	DC	0.6	0.14	0.15	0.06	**0.02**
		0.9	0.46	0.16	0.18	**0.00**
		1.2	4.30	0.84	0.57	**0.00**
CPU (Seconds)			0.70	0.82	0.31	2.57

2) Full factorial tests

In Table 3, the first and second columns correspond to the three problem parameters and their values based on which the analysis was done. Problem size is identified by the number of time periods in the planning horizon and the number of orders (T-N), which range from 10 to 100 periods and from 20 to 500 orders. The execution time of the solver when applied directly to the PP-OA-FDD formulation was limited to 900 Seconds. The last six columns in Table 3 present the average solution gap of two order-based relax-and-fix heuristics, two time-based relax-and-fix heuristics and the solver for a maximum CPU time of 900 Seconds (Column Solver 900s).

$$Gap' = 100 \times \frac{BestUB - Sol}{Sol}$$

Where Sol is the solution obtained using the solution approach and $BestUB$ is the best bound obtained using the solver. We also refer to Table 4 for a comparison of the average CPU times.

As it can be expected, from Table 3, the solver provides better quality solutions than the heuristics for very small problems though it requires much more CPU times on average. For medium and large instances, the time-based relax-and-fix heuristics (RF-T-5-3 and RF-T-10-8) outperform the solver in terms of solution quality while requiring much less CPU time. For example, for problems with $(T, N) = (100,200)$, the solver requires 630 seconds to reach an average gap of 4.05%, while RF-T-5-3 obtains solutions with an average gap of 1.99% in less than 12 Seconds on average.

For problems with a large number of orders, the order-based relax-and-fix heuristics are slower than the time-based heuristics as the number of sequences to be evaluated becomes large. The main reason behind constructing and evaluating several sequences is to search for sequences that would result in good quality solutions; yet, it can be seen from Table 3 that order-based heuristics results in relatively higher gaps when compared to the solver and time-based heuristics. Therefore, the time-based relaxed and fix heuristics are more suitable for solving this problem.

It can also be seen from Table 5 that the more customer due date flexibility is allowed (increasing Δ_i) and the tighter is the capacity (increasing DC), the harder is the problem to solve.

Table 3. Gaps (%) between the best bound and the best solution of the heuristics

T-N	RF-N-10-8	RF-N-15-10	RF-T-5-3	RF-T-10-8	Solver900
10-20	1.63	0.38	0.27	0.01	**0.00**
10-50	0.85	0.68	0.11	0.09	**0.05**
20-40	1.94	1.47	0.55	0.51	**0.35**
20-60	1.81	1.49	0.58	**0.51**	0.55
10-100	0.96	0.83	0.14	0.15	**0.09**
20-100	1.77	1.39	0.36	**0.31**	0.37
50-100	3.74	3.21	1.59	**1.44**	2.58
50-300	4.23	3.57	**0.64**	0.66	1.23
100-200	5.68	5.01	1.99	**1.89**	4.05
100-500	7.40	5.94	0.99	**0.83**	1.74

Table 4. Average CPU time (in Seconds) for different problem sizes.

T-N	RF-N-10-8	RF-N-15-10	RF-T-5-3	RF-T-10-8	Solver900
10-20	0.70	0.82	0.31	2.57	6.25
10-50	1.01	1.02	3.20	18.32	159.56
20-40	1.74	2.68	0.99	6.22	198.08
20-60	2.28	2.63	1.81	11.84	295.79
10-100	2.60	2.18	4.56	29.48	402.95
20-100	3.21	3.02	4.26	18.24	426.13
50-100	8.51	8.46	4.25	19.63	520.39
50-300	43.61	32.64	12.68	28.94	601.06
100-200	31.18	33.38	11.32	22.94	630.62
100-500	144.75	133.80	11.19	26.19	602.48

Table 5. Effect of Δ_i and DC on Gaps

		RF-N-10-8	RF-N-15-10	RF-T-5-3	RF-T-10-8	Solver900s
Δ_i	2	3.18	2.24	0.54	**0.37**	0.44
	4	3.06	2.50	0.74	**0.64**	1.05
	8	3.69	3.25	1.12	**1.11**	2.09
	12	4.77	4.38	**1.44**	1.47	3.23
DC	0.6	0.69	0.43	0.14	0.13	**0.00**
	0.9	2.30	1.82	0.54	**0.38**	0.49
	1.2	6.81	5.67	1.69	**1.61**	3.27

VI. CONCLUSION

Integrating production and sales decisions increases the competitiveness of manufacturing firms. In fact, by integrating production planning and order acceptance decisions companies can increase profit and in the same time customer satisfaction, by controlling delays and reducing them. Furthermore, negotiating flexible due dates allows companies to accept more orders and quote more reliable due dates to their customers. In this paper, we have proposed a mathematical programming formulation to model the integrated problem of production planning with load-dependent lead times, order acceptance, and flexible due dates. We quantified, through numerical experiments, the benefits of integration and due dates flexibility. For problems of realistic sizes, with a large number of planning periods and orders, the problem is very hard to solve in reasonable computational times. Therefore, two relax-and-fix heuristics have been developed to tackle this issue of dimensionality. Numerical results show that the time-based relax-and-fix heuristics outperform the order-based relax-and-fix heuristics and the direct application of a commercial solver as it provides better quality solutions in much less CPU times. Although the model presented in this paper considers more realistic behaviour of the capacity constraints, it still needs further improvements by considering other important issues related to production planning decisions such as setup costs, setup times and multi-products. Furthermore, faster solution approaches, which do not rely on the solution on integer linear programming problems need to be tackled and are currently under investigation.

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