Modeling and Simulation of Spatial Mechanism

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Abstract—**Spatial mechanisms have been an intensive area of research for over a decade, for their advantages such as good stiffness, large load capacity, and high accuracy, in comparison with the traditional serial mechanisms. The arm manipulators require an actuator for each axis, and a large number of degrees of freedom DOF, which make them expensive. In this case, the spatial mechanism can replace the serial manipulator. If we could design spatial mechanisms quickly and easily, there may be more applications using these mechanisms. The aerospace industry, sports training equipment and rehabilitation in the medical field already use some examples of these systems. In this paper, we analyze the proposed model of spatial mechanism based on kinematic and geometric criteria, the simulation results shows that varied values of the size of the fixed base and the length of the prismatic joint of the mechanism, the workspace takes different forms. This variation can be used to design the 3 UPU robot for specific applications.**

Keywords—*Spatial mechanism; Modeling; Simulation; Parallel Manipulator*

I. INTRODUCTION

Parallel manipulators have received a great attention in the last years for their potential and characteristics with respect to the serial manipulators. Indeed, these parallel manipulators exhibit a high nominal payload/weight ratio, a high positioning accuracy due to their inherent rigidity and a high dynamic performance, but a limited workspace and a low dexterous manipulability.

Different parallel manipulators with lower mobility and fewer than 6 DOF, also have attracted attention of many researchers. In the literature, many spatial parallel robots were designed and studied for specific tasks, such as the popular translational parallel manipulator Delta robot [1], it was generally used in pick & place tasks, and the 3 UPU parallel manipulator with its configurations, such as the 3 PUU. Parallel manipulators with pure translation are studied in-depth for their distinctive kinematic characteristics [2]–[11]. Tsai and Joshi [3], [4] presented a 3 UPU translational parallel manipulator, they compared its architecture with three legged parallel manipulator. The joints of the 3 UPU manipulator proposed by Tsai [3], must be in a particular configuration to maintain its translational motion, to do so, the outer revolute pair in the universal joint should be parallel. However, for the 3 UPU translational asymmetrical manipulator proposed by Lu Yi [6], the outer revolute pair in the universal joint are already

parallel due to the special arrangement in its legs, and it is a great advantage compared to the parallel manipulator proposed by Tsai.

The inverse kinematics of parallel manipulators is relatively simple and the direct kinematics is challenging. However, the direct kinematics of serial manipulators is simple while their inverse kinematics is quite complicated [12], [13]. The Jacobian matrix is developed then used to search for conditions that lead to singular configurations where the mobility of the manipulator instantly changes [14]. The condition number [15] of this matrix characterizes the dexterity of a robot manipulator at a given posture in the workplace, which is an important index to measure the performance of the mechanism. We also used an approximate approach to obtain graphically the workspace of the 3 UPU parallel manipulator [16].

This paper is organized as follows: Section 2 presents the structure of the 3 UPU manipulator, section 3 presents the inverse kinematic problem, section 4 of this paper shows the Jacobian matrix and the singular configurations, in section 5 workspace and dexterity visualizations are presented. Some concluding remarks are presented in section 6. All computations were done using Matlab R2014b.
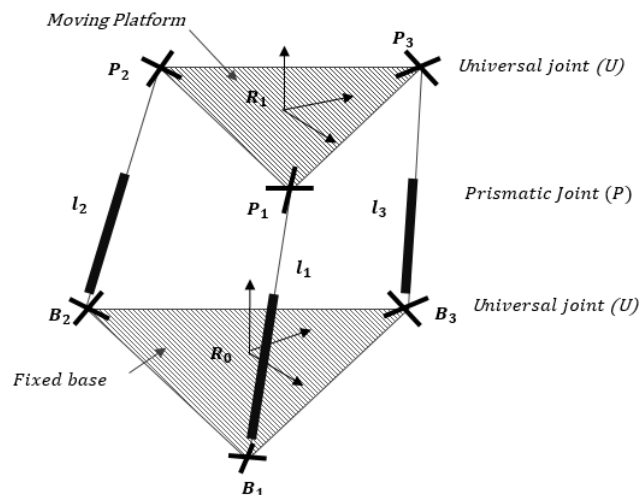
II. DESCRIPTION OF THE 3 UPU ROBOT

Fig. 1. 3 UPU parallel manipulator.

First of all an exact description of the mechanism is necessary. Referring to Fig. 1 the 3 UPU parallel manipulator is described as follows: In the base we have three points B_1, B_2 and B_3 which form and equilateral triangle. The frame $\Re_0$ is fixed in the base, its *yz* plane coincides with the plane formed by the triangle and its *z* axis passes through B_3. In the platform we have the same situation. An equilateral triangle P_1, P_2 and P_3, and its frame is denoted by $\Re_1$.

Now each pair of points B_i, P_i is interconnected by a limb of length l_i which is an independent serial leg of type UPU, where *U* and *P* stand for universal and prismatic joint, respectively. The universal joint *U* contains two intersecting revolute joints. The motion of the platform is controlled by extending or retracting the actuated prismatic joints [17].

The number of degrees of freedom DOF F is given by the *Chebychev-Grübler-Kutzbach* criterion.

Since we have eight links, six universal joints and three prismatic joints, the degrees of freedom of the 3 UPU parallel mechanism is calculated as :

$$F = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i \qquad (1)$$
$$F = 6(8 - 9 - 1) + (6 \times 2 + 3 \times 1) = 3$$

Where λ is the dimension of the space in which the mechanism is intended to work, *n* the number of links, *j* the number of joints and f_i represents the degrees of freedom associated with joint *i*.

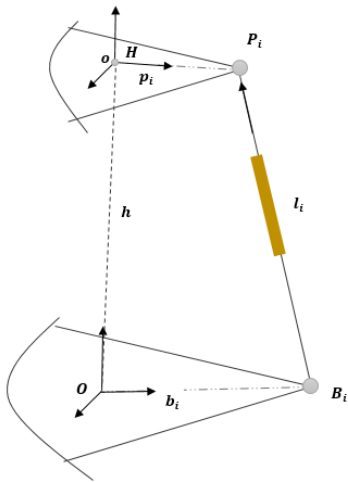### III. INVERSE KINEMATIC PROBLEM

Fig. 2. Geometric Modelling.

Fig. 2 shows a typical limb of the 3 UPU mechanism. The solution of the inverse kinematic problem is obtained geometrically. We express the vector h connecting the origin of the fixed base to the origin of the moving platform as follows:

$$h = b_i + l_i - p_i \qquad (2)$$

Where h, b_i, l_i and p_i are shown in Fig. 2, and *i*=1,2,3.

To simplify calculations, we include the vector u_i:

$$u_i = b_i - p_i \qquad (3)$$

Which lead to the set of equations:

$$l_i = h - u_i \qquad (4)$$

Dot multiplying (4) with itself yields:

$$l_i^2 = [h - u_i]^T [h - u_i] \qquad (5)$$

Finally:

$$l_i = \pm\sqrt{[h - u_i]^T [h - u_i]} \qquad \text{,for } i = 1, 2, 3 \qquad (6)$$

This equation corresponds to each given position of the platform, there are two possible solutions. The positive solution will be used.

IV. SINGULARITY OF THE 3 UPU MANIPULATOR

In this section the singularity of 3 UPU manipulator is analyzed. Singularities occur, when the Jacobian matrix becomes singular.
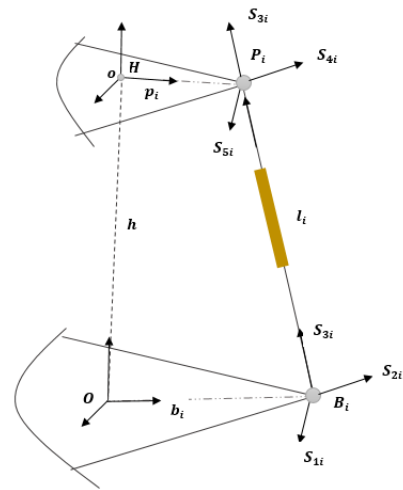

Fig. 3. A typical limb.

As shown in Fig. 3, the velocity of point P_i is given by:

$$V_{P_i} = \omega_i \times S_{3i} + \dot{l}_i S_{3i} \qquad (7)$$

Where ω_i and S_{ji} stand for the angular velocity of the *i*th limb with respect to the base and the unit vector pointing along the *j*th joint axis of the *i*th limb, respectively.

Now Dot multiplying both sides of (7) by S_{3i}, yields:

$$S_{3i}^{T} V_{P_i} = \dot{l}_i \qquad (8)$$

Since the moving platform has only translational motion, $\omega_i = 0$ and $V_{P_i} = V_H$.

When (8) is written for each limb, yields three scalar equations which can be expressed as follows:

$$J V_H = \dot{l} \qquad (9)$$

Where:

$$J = \begin{pmatrix} S_{31}^{T} \\ S_{32}^{T} \\ S_{33}^{T} \end{pmatrix} \qquad (10)$$

Is the Jacobian matrix and $\dot{l} = \begin{bmatrix} \dot{l}_1, \dot{l}_2, \dot{l}_3 \end{bmatrix}^T$ is the vector of joint velocities.

The analysis of (9) shows that singularities occurs when the Jacobian matrix J becomes singular. Each row of the matrix corresponds to a direction of a limb, the mechanism then will be in singular configuration if the three unit vectors S_{3i}, $i = 1, 2, 3$ become linearly dependent. Thus some cases are possible:
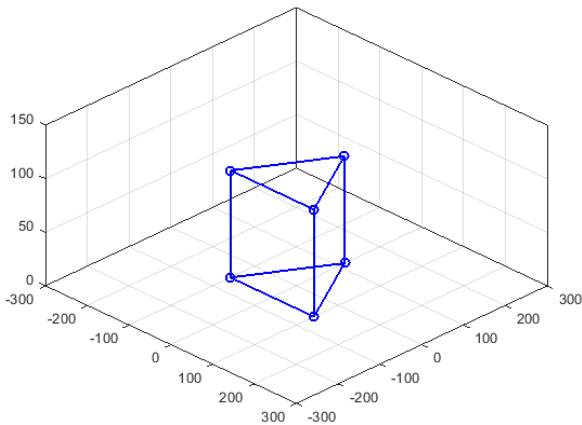

Fig. 4.   Singular configuration – Parallel limbs.

The first case in Fig. 4 occurs if all three limbs are parallel. To allow this configuration, the geometry of the moving platform and the fixed base must be identical and $l_1 = l_2 = l_3$.

The second possibility Fig. 5 appears when the three limbs are coplanar, or when they are pointing toward the center of the moving platform.
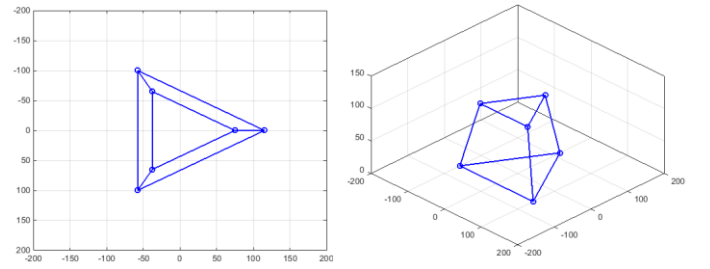

Fig. 5.   Singular configuration – Coplanar limbs.

V.   DEXTERITY AND WORKSPACE ANALYSIS

A.   *Dexterity of 3 UPU manipulator*

Dexterity is an important issue for design, trajectory planning, and control of robots. The dexterity of a manipulator is defined as the ability of the manipulator to move its platform in all directions. The farther a manipulator is from a singular configuration the better its dexterity.

It can be seen from Fig. 6 that the dexterity of the mechanism progressively decreases when the height of the moving platform increases. This is consistent with the singularity analysis. By increasing the height of the end-effector, the 3 limbs comes close to the singular configuration where they are parallel. The opposite is also true.
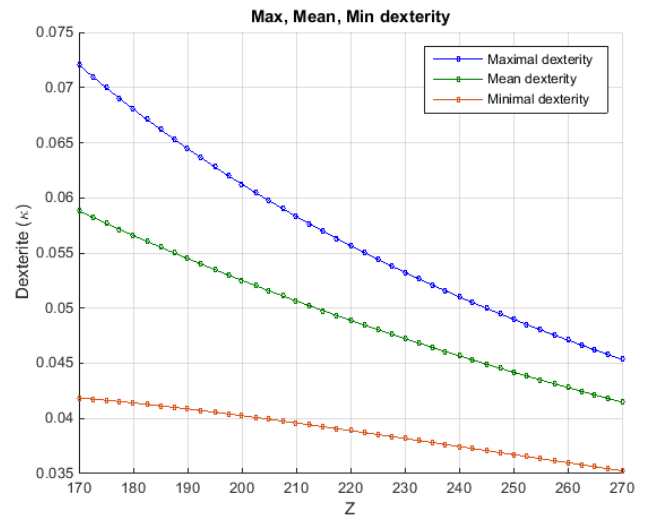

Fig. 6.   Dexterity analysis of 3 UPU mechanism depending on height.

B. Workspace of 3 UPU manipulator

Because of the translational architecture of 3 UPU parallel manipulator, its workspace contains only points that are reachable by the moving platform. This combination of points of the moving platform center is viewed as the 3 UPU robots workspace. To improve the applications of parallel manipulators, it is essential to analyze the form and volume of workspace. In this part, we studied the variation of the 3 UPU parameters and their impact on the reachable workspace.

C. Algorithms

In this section an approximate method is used to obtain graphically the workspace of the 3 UPU manipulator. The workspace is obtained by identifying all reachable positions by the moving platform with respect to the joint limits. A discretization function is therefore used in this case. This function consists in discretizing the three dimensional space, solving the inverse kinematic problem at each point to provide a sequence of points (p_x, p_y, p_z), and searching the boundaries of the workspace. The whole workspace and its boundary are calculated quantitatively for the 3 UPU mechanism.

As demonstrated in Fig. 7, the upper and lower surfaces of the workspace are traced from selected points with default parameters.

D. Simulation results and discussions

It can be seen from Fig. 3 that the parameters of the 3 UPU translational parallel manipulator consist of the sizes of the fixed base " b " and the moving platform " p ", and the two limits of the three limbs " l ". To compare the results, when one of the parameters are changed, we have to fix other parameters when they are not varied, which are the default parameters, let the size of the base $b = 200$, the platform $p = 170$, the maximum limit $l_{max} = 270$, and the minimum limit $l_{min} = 170$.
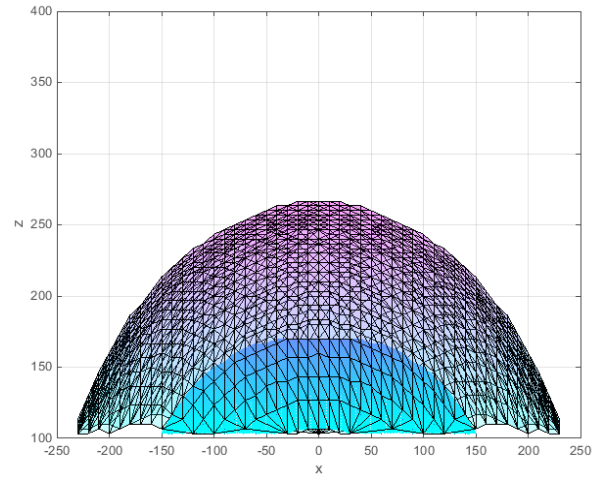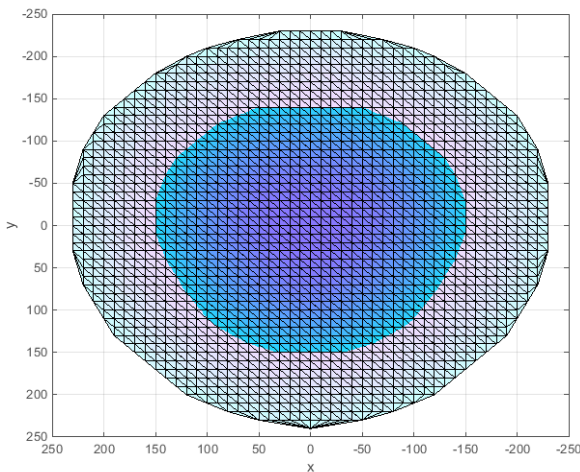

(b) Projection in x-z plane
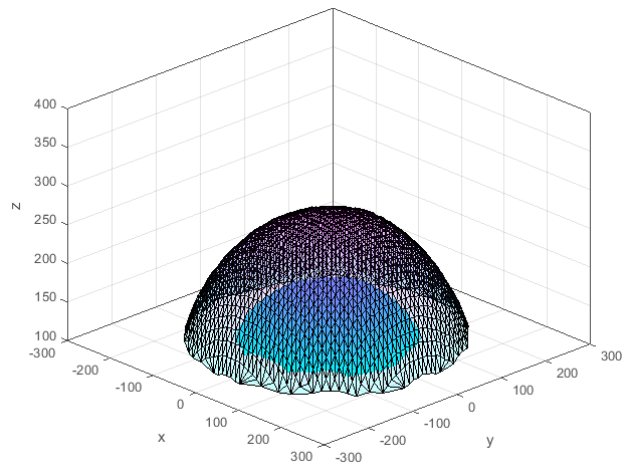
(c) Three dimensional workspace

Fig. 7. Workspace volume: (a) Projection in xy plane – (b) Projection in xz plane – (c) Three dimensional workspace.

The workspace of the 3 UPU manipulator with the default parameters is presented in Fig. 7, it consists of a bigger upper dome that cover a smaller lower one.
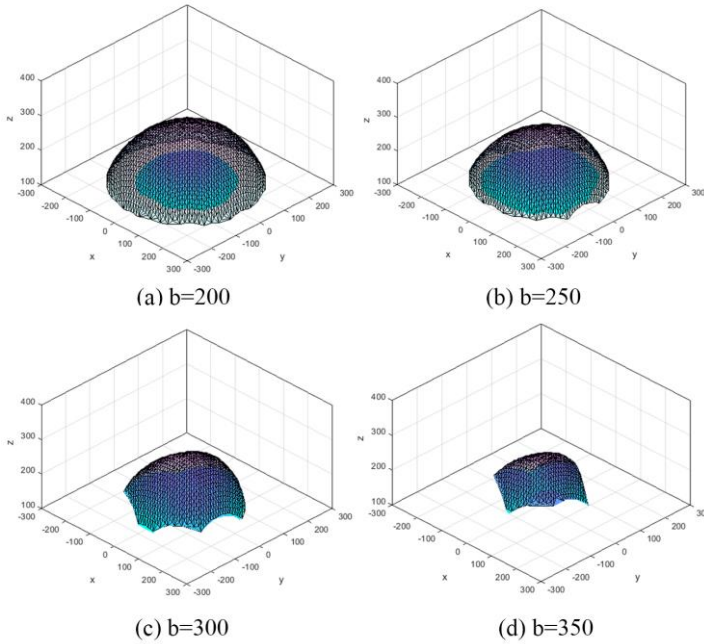

(a) Projection in x-y plane

Fig. 8. Workspace form versus the fixed base size

Fig. 8 shows four workspace illustrations, which present the impact of the fixed base size on the workspace form. Increasing the size of the base, the form of the workspace is reduced considerably.
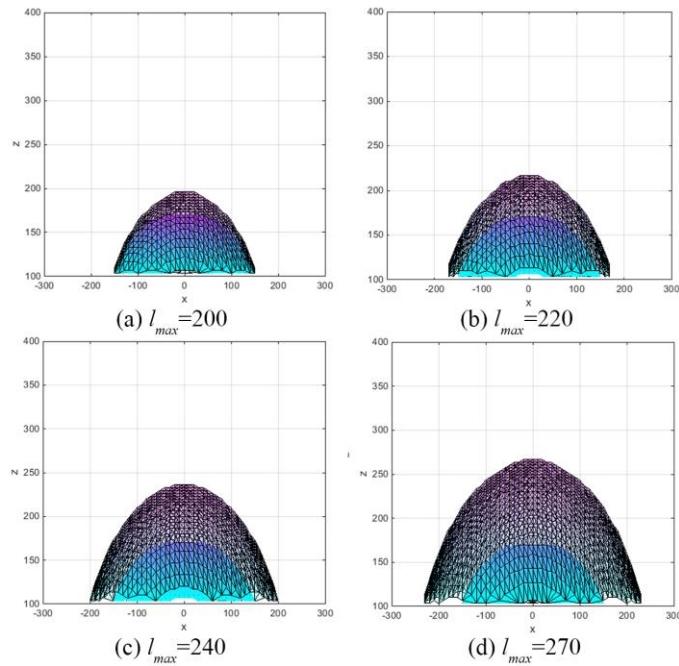

Fig. 9. Workspace form versus the maximum limit

Besides that, Fig. 9 illustrates four workspace projections on z-x plane, which indicate the impact of the increasing of the maximum limit of the prismatic joint on the workspace form. By increasing the maximum limit the upper dome becomes relatively large.

Finally, gathering all the simulation results, we can conclude that the fixed base size has the most impact on the workspace form besides the influence of the maximum limit on the workspace volume.

VI. CONCLUSION

In this paper, we present the spatial 3 UPU parallel manipulator. The solution of inverse kinematic problem was found geometrically. The Jacobian matrix is used to analyze the singular configurations of the 3 UPU robot. We also represent the evaluation index of dexterity and the workspace visualizations. By changing the size of the fixed base and the length of the prismatic joint of the mechanism, the workspace takes different forms. This variation can be optimized and used to design the 3 UPU robot for specific applications.

References

[1] R. Clavel, "DELTA, a Fast Robot with Parallel Geometry," Jan. 1988.

[2] J. P. Merlet, Parallel Robots. Springer Science & Business Media, 2012.

[3] L.-W. Tsai and S. Joshi, "Kinematics and Optimization of a Spatial 3 UPU Parallel Manipulator," J. Mech. Des., vol. 122, no. 4, p. 439, Dec. 2000.

[4] S. Joshi, "A comparison study of two 3-DOF parallel manipulators: one with three and the other with four supporting legs," in Proceedings 2002 IEEE International Conference on Robotics and Automation (Cat. No.02CH37292), 2002, vol. 4, pp. 3690–3697.

[5] R. Di Gregorio, "Kinematics of the Translational 3-URC Mechanism," J. Mech. Des., vol. 126, no. 6, p. 1113, Nov. 2004.

[6] Y. Lu and B. Hu, "Analysis of kinematics and solution of active/constrained forces of asymmetric 2UPU+X parallel manipulators," Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci., vol. 220, no. 12, pp. 1819–1830, Dec. 2006.

[7] K. Zhou, J.-S. Zhao, Z.-Y. Tan, and D.-Z. Mao, "The kinematics study of a class of spatial parallel mechanism with fewer degrees of freedom," Int. J. Adv. Manuf. Technol., vol. 25, no. 9–10, pp. 972–978, Nov. 2004.

[8] W. Li, F. Gao, and J. Zhang, "R-CUBE, a decoupled parallel manipulator only with revolute joints," Mech. Mach. Theory, vol. 40, no. 4, pp. 467–473, Apr. 2005.

[9] D. Kim and W. K. Chung, "Kinematic Condition Analysis of Three-DOF Pure Translational Parallel Manipulators," J. Mech. Des., vol. 125, no. 2, p. 323, Jun. 2003.

[10] M. Carricato and V. Parenti-Castelli, "Singularity-Free Fully-Isotropic Translational Parallel Mechanisms," Int. J. Rob. Res., vol. 21, no. 2, pp. 161–174, Feb. 2002.

[11] X. Kong and C. M. Gosselin, "Type Synthesis of 3-DOF Translational Parallel Manipulators Based on Screw Theory," J. Mech. Des., vol. 126, no. 1, p. 83, Jan. 2004.

[12] L.-W. TSAI, "Kinematics of A Three-Dof Platform with Three Extensible Limbs," Recent Adv. Robot Kinemat., Jan. 1996.

[13] R. Di Gregorio and V. Parenti Castelli, "A translational 3-DoF parallel manipulator," Adv. Robot Kinemat. Anal. Control, vol. 3, pp. 49–58, 1998.

[14] I. A. Bonev, D. Zlatanov, and C. M. Gosselin, "Singularity Analysis of 3-DOF Planar Parallel Mechanisms via Screw Theory," J. Mech. Des., vol. 125, no. 3, p. 573, Sep. 2003.

[15] C. Gosselin and J. Angeles, "A Global Performance Index for the Kinematic Optimization of Robotic Manipulators," J. Mech. Des., vol. 113, no. 3, p. 220, 1991.

[16] C. Gosselin, "Determination of the Workspace of 6-DOF Parallel Manipulators," J. Mech. Des., vol. 112, no. 3, p. 331, 1990.

[17] W. T. Appleberry, "Anti-rotation positioning mechanism." 20-Oct-1992.