

Vehicle Dynamics and Steering Angle Estimation Using a Virtual Sensor

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Abstract—Vehicle stability control systems called ESP, vehicle dynamics control (VDC), yaw stability control (YSC), and so forth are important active safety systems used for maintaining lateral stability of the vehicles under such adverse conditions. This paper investigates the notion of virtual sensing which is a promising concept for yaw stability control and is an attractive option for vehicle manufactures as it reduces sensor cost, maintenance, and machine downtime.

A frequent situation in automotive control applications is when an unknown input needs to be estimated from available state measurements. The virtual sensor proposed use measurements of lateral acceleration, steering angle as unknown input signals and provide the yaw rate angle estimate as output. Stability conditions of this virtual sensor are given in terms of Linear Matrix Inequalities (LMI). To illustrate the proposed methodology, a linear bicycle model is considered. The observer(virtual sensor) is confronted to data issued from the Callas vehicle simulator.

Index Terms—Virtual Sensor, State Space, Modeling, Vehicle Dynamics, Yaw-rate, Steering Angle.

I. INTRODUCTION

Road vehicle yaw stability control systems like electronic stability program (ESP) are very important for the safety of the driver and passengers during extreme lateral maneuvers or during lateral maneuvers under adverse environmental conditions like driving on snow or ice, sudden tire pressure loss, or sudden side wind. These mechatronics systems used for maintaining lateral stability of the vehicle, based their calculations on measurements from sensors (accelerometers, gyrometers, steering wheel angle, wheels position sensors . . .). Nevertheless, some variables are actually unreachable to measure (longitudinal acceleration, side slip angle, yaw rate, tire/road forces. . .) Vehicle yaw rate is the key parameter that needs to be known by a yaw stability control system. In this paper, yaw rate is estimated using a virtual sensor

Some observers can be done to estimate these variables [1], [2]. Researches are conducted to develop low cost sensors to acquire tire/road forces [3], [4]. Such instrumentation has been

considered especially by builders automotive [5], of Roulementier [6] by SNR-NTN company [7], [8], with partners [9], [10] or SKF company [11] of have show the feasibility of the integration of forces measurements in a future generation of affordable wheel bearing.

Since the experience of most drivers is limited to driving within the linear range of vehicle handling behavior, It is generally considered desirable to reduce the difference between the normal vehicle behavior and that at the limit. This improves the chances of a typical driver to maintain control of the vehicle in emergency situations. This goal can be accomplished by active chassis control systems such as brake control systems or active rear wheel steer. The purpose of control is to bring the vehicle yaw rate response and/or the vehicle slip angle into conformance with the desired yaw rate and/or slip angle.

With this objective, this article presents a virtual sensor to estimate simultaneously vehicle side slip angle, yaw rate and the steering angle considered as an unknown input. The yaw rate is considered as the only available measure.

The estimated state and unknown input can then be used as input of actual ADAS (Advanced driver assistance systems). Several studies were conducted to estimate the unknown inputs the linear dynamical systems [12], [13], [14], [15] and [16]. Edwards and Spurgeon have proposed two methods based on sliding mode observers for detect and estimate the sensor fault [17].

The article of [18] uses a bicycle model to estimate the center of gravity of a motor vehicle. [19] present this model, a four wheel steering, for control side slip angle and yaw rate.

In this first study, a vehicle bicycle model [20] is used to design the observers. The longitudinal speed of the vehicle is supposed to be constant, lateral tires forces linear with respect to side slip angle and sprung mass movements are neglected. This kind of model is valid for low lateral dynamics ($|\dot{y}| < 0,4g$). Observer is designed using the "unknown input with unknown input is independent of the output" framework [21], [22] and [23]. The gain of the observer is computed by solving linear matrix inequalities [24] and [25]. The organization of the rest of the paper is as follows. In Section 2 and its subsections presents the vehicle modeling and dynamic virtual sensor design is explained. Simulation results obtained using the virtual sensor are given, observer is tested

and confronted to data issued from the vehicle simulator Callas distributed by Oktal company In Section 3, and the paper ends with conclusions. Notations are explained in section VI.

II. VEHICLE DYNAMICS MODEL

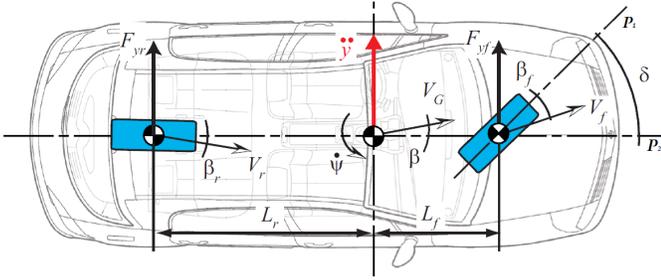


Fig. 1. bicycle Model [26]

Lateral vehicle dynamics has been studied since the 1950s. In 1956, Segel [20] presented a vehicle model with three degrees of freedom in order to describe lateral movements including roll and yaw. If roll movement is neglected, a simple model known as the bicycle model is obtained. This model is currently used for studies of lateral vehicle dynamics (yaw and sideslip).

The bicycle model (1) used in this paper is obtained by application of the fundamental principal of dynamics. Variables are presented on Fig. 1. Inputs of model are lateral tire/road forces projected onto the vehicle frame, The equations of this model are described as follows:.

$$\begin{cases} \dot{\beta} &= (F_{yf} + F_{yr}) / (V_G M_V) - \dot{\psi} \\ \dot{\psi} &= (L_f F_{yf} - L_r F_{yr}) / I_{zz} \end{cases} \quad (1)$$

Using the assumptions of low lateral dynamics, small steering and side slip angle, lateral forces are proportional to the tires side slip angles. They can be written as:

$$\begin{cases} F_{yf} &= D_f \beta_f \\ F_{yr} &= D_r \beta_r \end{cases} \quad (2)$$

Tires side slip angles (front and rear) can be approximated using kinematic relations. The side slip angle of front tires β_r is between P_1 and V_f . The side slip angle of rear tires β_r is between P_2 and V_r .

$$\begin{cases} \beta_f &= \delta - \beta - \frac{L_f \dot{\psi}}{V_G} \\ \beta_r &= -\beta + \frac{L_r \dot{\psi}}{V_G} \end{cases} \quad (3)$$

Linear tire forces are:

$$\begin{cases} F_{yf} &= D_f (\delta - \beta - L_f \frac{\dot{\psi}}{V_G}) \\ F_{yr} &= D_r (-\beta + \frac{L_r \dot{\psi}}{V_G}) \end{cases} \quad (4)$$

III. LINEAR UNKNOWN INPUT OBSERVER WITH UNKNOWN INPUT IS INDEPENDENT OF THE OUTPUT

The aims of the observer presented in this paper are to calculate the vehicle lateral state variables and the steering angle that is not measured. Assuming that the only available measure is the yaw rate and we want to estimate center of gravity side slip angle, yaw rate (states) and the steering angle (unknown input). The proposed observer will be an linear Unknown Input Observer with Unknown Input is Independent of The Output UIO.

A. State-space model

By substituting F_{yf} and F_{yr} (4) in equation (1), the dynamics of the bicycle model can be written as a linear system with unknown input:

$$\Sigma \begin{cases} \dot{x} &= Ax + R\delta \\ y &= Cx \end{cases} \quad (5)$$

In this expression,

$x = \begin{pmatrix} \beta \\ \dot{\psi} \end{pmatrix}$ is the state vector, δ is the unknown input and $y = \dot{\psi}$ is the output vector. A is the state matrix, R is the input matrix associated with the unknown input, C is the observation matrix.

$$A = \begin{pmatrix} \frac{-D_f - D_r}{M_V V_G} & \frac{L_r D_r - L_f D_f}{M_V V_G^2} - 1 \\ \frac{L_r D_r - L_f D_f}{I_{zz}} & \frac{-L_r^2 D_r - L_f^2 D_f}{I_{zz} V_G} \end{pmatrix}$$

$$R = \begin{pmatrix} \frac{D_f}{M_V V_G} \\ \frac{L_f D_f}{I_{zz}} \end{pmatrix}$$

$$C = (0 \quad 1)$$

B. Observer design

It often happens that all state variables of a system are not accessible to measurement or input not measurable because of very expensive sensors or absence of technology. The idea is to rebuild the state and the input not measurable from information available that is to say the output and known inputs.

An unknown input observer (UIO) estimates simultaneously the state x and the unknown input δ by using the available measurements that constitute the output y (Fig. 2).

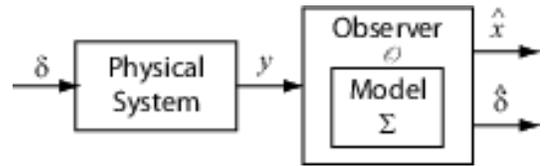


Fig. 2. Unknown inputs observer principle

For the linear time invariant (LTI) systems, a lot of observation techniques can be applied (Kalman filter, Luenberger observer, sliding mode observer, ...). To be able to characterize the robustness performances, the gains of observer UIO are obtained by solving a Linear Matrices Inequalities problem.

The full order observer ([21] and [22]) for system (5) can be expressed as:

The system (5) respects the following assumptions :

- The unknown inputs are not involved in the measure.
- The number of unknown inputs is less or equal to the number of measures.
- The measure is one of the state variables..
- The matrix R is full column rank $Rank(R) = 1$.

An unknown input observer exists if and only if the system (1) is detectable (observable) and the number of unknown inputs less than or equal to the number of measures [23].

The observability matrix of this new system is written as:

$$O = \begin{pmatrix} C \\ CA \end{pmatrix}$$

The observability matrix (O) is full rank ($rank(O) = 2$), so the system (5) is observable.

It is noted that $(L_r D_r \neq L_f D_f)$ is always available.

The numerical value of the determinant of this matrix in the case of the studied vehicle is -32.3171.

C. Observer design UIO with Unknown Input is Independent of The Output

The full order observer ([23], [27], [12] and [22]) for system (1) can be expressed as:

$$obs \begin{cases} \dot{z} = Nz + Ly \\ \hat{x} = z - Ey \end{cases} \quad (6)$$

Where z is the dynamic of the observer. $\hat{x} = (\beta \ \psi)^T$ is the observed state. N , L and E are unknown matrices which must be determined such that \hat{x} will asymptotically converge to x .

Define the observer reconstruction error by:

$$e = x - \hat{x} = z - x - Ey$$

Then, the dynamic of this observer error is:

$$\dot{e} = (I + EC)(Ax + R\delta) - (N\hat{x} + (LC + NEC)x)$$

Let

$$P = I + EC \quad (7)$$

Then, the observation error dynamic is

$$\dot{e} = Ne + PR\delta + (PA - NP - LC)x \quad (8)$$

The estimation error converges asymptotically to 0 if and only if the matrices N , L and E are choose to satisfy the following conditions:

$$\begin{cases} N \text{ is a stable matrix (Hurwitz matrix)} \\ PR = (I + EC)R = 0 \\ LC = PA - NP \end{cases} \quad (9)$$

Equation (8) reduces to the homogeneous equation

$$\dot{e} = Ne \quad (10)$$

The following paragraphs present the determination of matrices N , E and L satisfying the constraints (9).

1) Matrix E:

This matrix E is calculated using the second equation of system (9), the numerical solution of this equation is based on the calculation of the pseudo inverse of matrix CR , a possible solution is:

$$E = -R(CR)^T[(CR)(CR)^T]^{-1} \quad (11)$$

From equations (7) and (11), the matrix P can then be expressed as:

$$P = I - R(CR)^T[(CR)(CR)^T]^{-1}C \quad (12)$$

2) Matrix L:

The matrix L is determined from the third equation (9).

$$LC - PA + NP = 0 \Rightarrow N(I + EC) + LC - PA = 0$$

Then

$$N = PA - KC \quad (13)$$

With

$$K = (PA - KC)E + L \quad (14)$$

Therefore, the matrix L is given by:

$$L = K(I + CE) - PAE \quad (15)$$

3) Matrix N:

Determining the matrix L requires the determination of the matrix N (matrix K).

The Matrix L is chosen so that the observation error (10) is asymptotically stable.

Using the Lyapunov theorem, the convergence of the observer is guaranteed if there exists a symmetric positive matrix X that the Lyapunov function $V(e) = e^T X e$ presents the following properties:

$$\forall e \neq 0 \begin{cases} V(e) > 0 \\ \dot{V}(e) < 0 \end{cases}$$

This can be reformulated using equation (10):

$$\begin{cases} X > 0 \\ N^T X + XN < 0 \end{cases}$$

Applying the Schur complement [24], [25]

$$\begin{pmatrix} -X & 0 \\ 0 & N^T X + XN \end{pmatrix} < 0$$

Since $N = PA - KC$ (equation (13)), which allows:

$$\begin{pmatrix} -X & 0 \\ 0 & (PA - KC)^T X + X(PA - KC) \end{pmatrix} < 0 \quad (16)$$

Noticing that the inequality (16) is bilinear compared to the variables K and X . A resolution method is to conduct a change of variable:

$$W = XK$$

The linear matrix inequalities system (16) can be written as:

$$\begin{pmatrix} -X & 0 \\ 0 & (PA)^T X + X(PA) - (C^T W^T - WC) \end{pmatrix} < 0$$

The gain matrix is obtained by first solving the LMI with respect to X and W . In a second time, K is determined by:

$$K = X^{-1}W \quad (17)$$

Then, the matrices L and N is written:

$$\begin{cases} N &= PA - X^{-1}WC \\ L &= X^{-1}W(I + EC) - PAE \end{cases} \quad (18)$$

4) Estimation of unknown input:

The unknown input δ can be expressed from the output of system (5):

$$\delta = R^+(\dot{x} - Ax) \quad (19)$$

If state x is known, we can estimate δ (steering angle) from equations (19). What gives, when we replace \hat{x} (6) by its expression:

$$\delta = R^+(\dot{z} - E\dot{y} - A\hat{x})$$

In practice the derivative can be approximated by the formula:

$$sY(s) \simeq \frac{s}{1 + \tau s} Y(s)$$

Where s the Laplace operator and τ is a real number chosen to have $\dot{y} \simeq sy$, so τ must be small compared to the time constraint of the system to determine.

5) In summary:

To summaries, the observer linear Unknown Input Observer with Unknown Input is Independent of The Output UIO allows calculating simultaneously vehicle side slip angle, yaw rate and steering angle (unknown input) by using front lateral tire forces. It's written

$$UIO \begin{cases} \dot{z} = Nz + L\dot{\psi} \\ \begin{pmatrix} \hat{\beta} \\ \dot{\psi} \end{pmatrix} = z - E\dot{\psi} \\ \hat{\delta} = R^+ \left(\dot{z} - E \frac{s}{1 + \tau s} \dot{\psi} - A \begin{pmatrix} \hat{\beta} \\ \dot{\psi} \end{pmatrix} \right) \end{cases} \quad (20)$$

IV. NUMERICAL RESULTS

A. Vehicle simulator

Callas is a realistic vehicle simulator software distributed by Oktal company (<http://www.oktal.fr>). According to [28], it has been validated by car manufacturers and French research institutions including INRETS ("Institut national de recherche sur les transports et leur sécurité"). Callas is a physical based model that takes into account numerous aspects among which vertical dynamics (suspension, tires, road profile), kinematics, elasto-kinematics, tire adhesion, aerodynamics, ...

B. Simulation environment

Data used to evaluate observer performance are issued from the Callas simulator and the simulation environment is Matlab-Simulink.

Parameterization of open-loop model (5) used in the construction of observers is verified using a linear stationary bicycle model, whose input is the steering angle measured and considered as known δ_m . This model (5) is called OLM in the paper.

The results of observer UIO (20) is compared with the outputs of the simulator CALLAS.

The observer has been tested in the case of an ISO double lane change at 40km/h and 90km/h. Initial conditions of the model and observers states are zero. The route to the lateral strain of the vehicle is given in Fig. 3.

C. Vehicle parameters

- Front tire cornering stiffness: $D_f = 96000 [N.rad^{-1}]$.
- Rear tire cornering stiffness: $D_r = 69500 * 2 [N.rad^{-1}]$.
- Half-wheelbase front: $L_f = 1.1824 [m]$.
- Half-wheelbase rear: $L_r = 1.5176 [m]$.
- Total vehicle mass: $M_V = 1683 [kg]$.
- Yaw inertia: $I_{zz} = 3015 [kg.m^2]$.

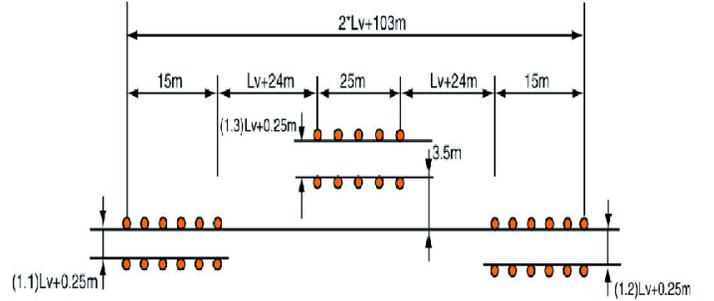


Fig. 3. Chicane ISO: positioning corridors.

D. Yaw Rate as "measure"

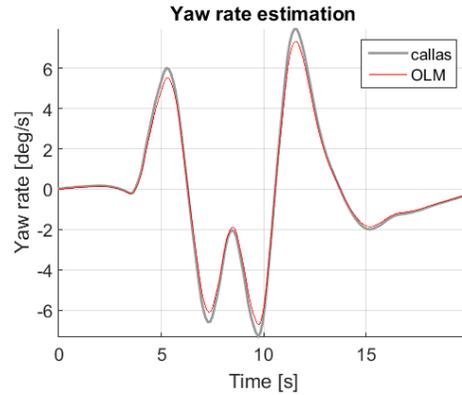


Fig. 4. Yaw Rate for ISO double lane change 40km/h. Callas simulation compared with open-loop model (5).

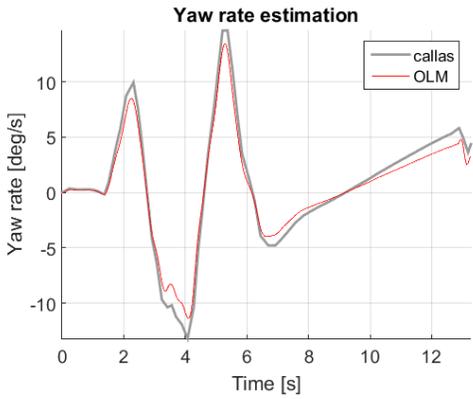


Fig. 5. YawRate for ISO double lane change 90km/h. Callas simulation compared with open-loop model (5).

yaw rate of the vehicle (bicycle model) is given in Fig. 4 for the test at 40km/h and in Fig. 5 for the test at 90km/h.

These figures illustrate the validity domain of the linear bicycle model. At 40km/h, the lateral acceleration is low and the model is representative. On the other hand, at 90km/h, tires are positioned in their transient zone, model is less representative. For this test, the maximum lateral acceleration is $5m.s^{-2}$.

E. ISO Double lane change 40km/h

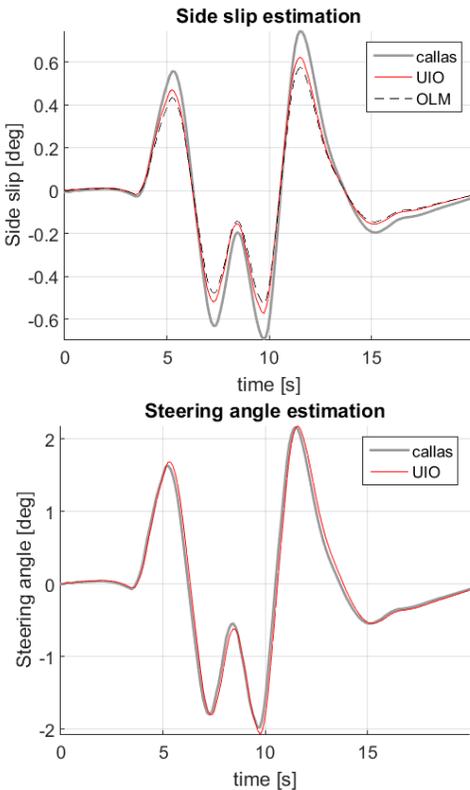


Fig. 6. Observer : side slip angle and steering angle estimations for ISO double lane change 40km/h. Callas simulation compared with unknown input observers UIO (20). Recall that the steering angle is not measured.

Estimations of vehicle side slip angle, yaw rate (state variables) computed by observer UIO are compared to the values issued from the Callas simulator used here as reference. The estimation of the steering angle (the unknown input) is compared to the reference computed by the virtual driver of the simulator. The side slip angle, yaw rate and unknown steering is treated on Fig. 6. This indicates that not measured states and the unknown input are correctly calculated by observer based on yaw rate measures. However, it is noteworthy that observer (and the underlying open-loop model) underestimated the side slip angle. It is important to remind here that the steering angle is supposed not measured but is well computed.

F. ISO Double lane change 90km/h

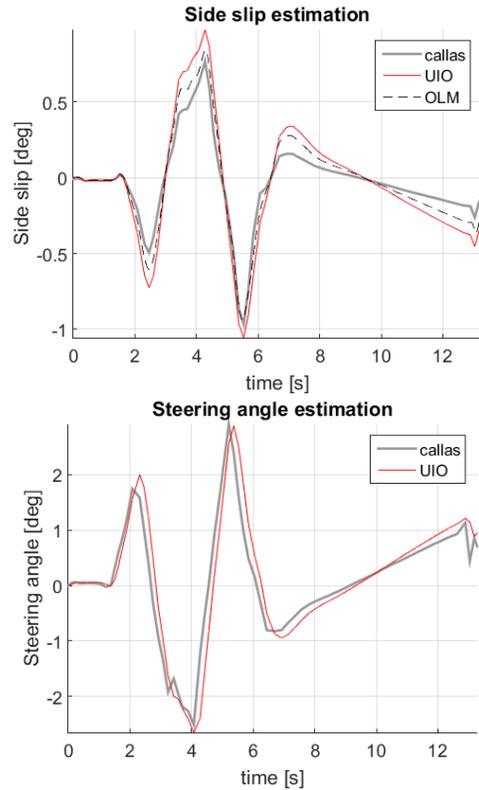


Fig. 7. Observer : side slip angle and steering angle estimations for ISO double lane change 90[km/h]. Callas simulation compared with unknown input observers UIO (20). Recall that the steering angle is not measured.

For the ISO double lane change test at 90km/h, vehicle side slip angle is presented and unknown steering angle on Fig. 7 This shows the good performances for observer for all estimations. The used tire model (linear and constant cornering stiffness) reached its representative limits. This over estimation is amplified by observer. Performances on yaw rate estimation of their part are enhanced. Computation of the unknown steering is quite good.

G. Errors of estimates

The error of the observer UIO for estimate side slip angle, yaw rate and steering angle of an ISO double lane change at 40 and 90km/h data by Fig. 8 and Fig. 9.

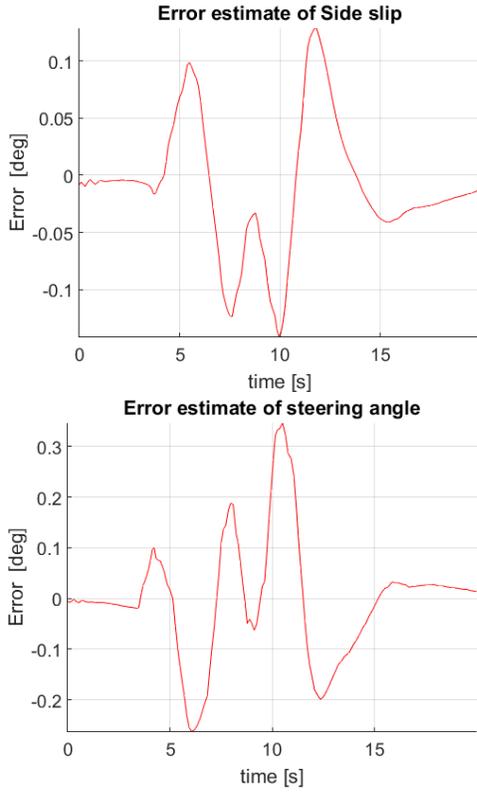


Fig. 8. Errors of Observer : Errors estimates of side slip angle and steering angle for ISO double lane change 40[km/h]. Recall that the steering angle is not measured.

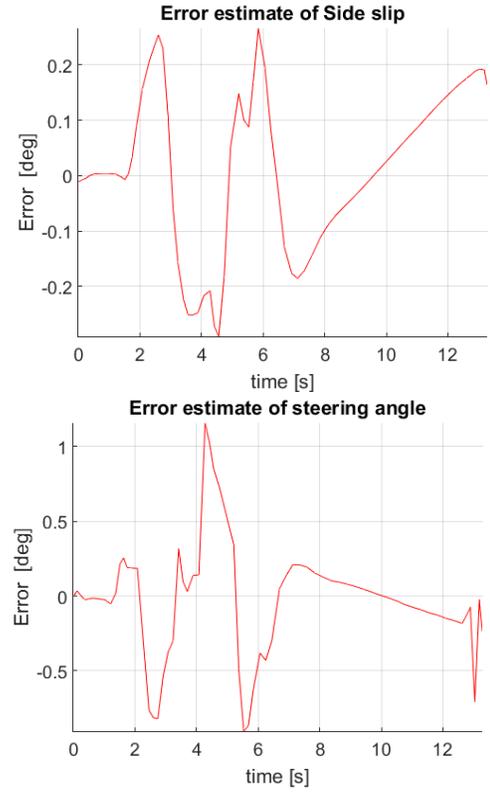


Fig. 9. Errors of Observer: Errors estimates of side slip angle and steering angle for ISO double lane change 90[km/h]. Recall that the steering angle is not measured.

1) *ISO Double lane change 40km/h*: The maximum error of estimation of the side slip angle is of order $\simeq 0.15[deg]$, obtained to $t = 11.97[s]$, established by the observer. The maximum error of estimation of the steering angle is of order $\simeq 0.33[deg]$, obtained to $t = 11.13[s]$, established by the observer. The comparison of these errors, issued by the observers to unknown inputs, shows that we obtain a better estimate of states (side slip angle, yaw rate) where the observer UIO and the best estimate of the unknown inputs (steering angle) is obtained by the observer from UIO.

2) *ISO Double lane change 90km/h*: 1.2[deg] obtained to $t = 12[s]$ is maximum error of estimation as the side slip angle (β) issued by observer. The maximum error of estimation of the steering angle (δ) is of order $\simeq 0.66[deg]$, obtained to $t = 10.5[s]$, established by the observer UIO. The comparison of these errors, issued by the observers to unknown inputs, shows that we obtain a better estimate with observer UIO for speed yaw.

V. CONCLUSION

In this paper we have presented a steering angle estimator based on a recently proposed direct approach for the design of virtual sensors. Current virtual sensors for steering angle estimation based on yaw stability control are used in the electronic stability control (ESC) system for diagnostic International Journal of Vehicular Technology. The obtained steering virtual sensor has been tested on number of different manoeuvres,

including double lane change using a linear bicycle model at constant speed and a linear tire model have been used to characterize the vehicle lateral dynamics. This model is valid only if the lateral dynamics are low. But, the LTI framework makes easier the design of unknown input observers. This kind of observers allows not only compute variables usually measured (yaw rate and steering wheel angle) but also the side slip angle. The obtained results showed that the proposed linear estimator is able to provide a good steering angle estimation in a large range of operation and the proposed yaw stability controller is expected to improve yaw stability within linear tire regions. Gain of the observer has been computed by solving linear matrix inequalities. This methodology provides a robust design (in a future work). Finally, the designed virtual sensor was tested using data issued from the Callas vehicle simulator and the results were found to be quite satisfactory.

VI. NOTATIONS

D_f : Front tire cornering stiffness [$N.rad^{-1}$]
 D_r : Rear tire cornering stiffness [$N.rad^{-1}$]
 L_f : Half-wheelbase front [m]
 L_r : Half-wheelbase rear [m]
 I_{zz} : Yaw inertia [$kg.m^2$]
 M_V : Total vehicle mass [kg]
 V_G : Norm of speed of center of gravity [$m.s^{-1}$]
 $\dot{\psi}$: Yaw rate [$rad.s^{-1}$]
 F_{yf} : Front lateral tire force [N]

F_{yr} : Rear lateral tire force [N]
 β : Center of gravity side slip angle [rad]
 β_f : Front wheel side slip angle [rad]
 β_r : Rear wheel side slip angle [rad]
 δ : Front wheel steering angle [rad]
 σ_f : Front relaxation length [m]
 σ_r : Rear relaxation length [m]
 \hat{x} : Estimation of x

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