Simultaneous design of a product and its supply chain integrating reverse logistic operations: An optimization model

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Abstract—**This paper deals with a recent approach that tackles the product and the supply chain design issues at the same time. On one hand, supply chain constraints must be integrated into product design phase. On the other hand, product specificities are considered while determining supply chain structure. Moreover, reverse activities are integrated to recapture value of used products and also for ecological purpose.**

We study the case of an existing product's redesign. The design change affects either the components or the process of the product or both of them at once. Therefore, we prioritize the optimization of supplying, production and recycling costs. For the product redesign, bill of materials is considered. For the supply chain configuration, suppliers' selection, assembling process choice, recycling centers' location are considered as well as sub-contractors selection.

At product design phase, the design team offers several design alternatives. The aim of this work is to determine simultaneously the optimal redesign alternative accompanied with its optimal supply chain configuration integrating recycling activity. For this purpose, a mixed integer linear programming (MILP) formulation is proposed. Thus, the product redesign and the supply chain configuration chosen should optimize supply, transportation, storage and recycling of components on one hand and production, outsourcing, technologies implementation and product's recycling on the other hand. Since the product could be returned during its entire life cycle, time periods in the MILP formulation are considered to be product life cycle phases. This consideration gave a dynamic aspect to our model.

Keywords—Supply chain design; Product redesign; Supply chain optimization; reverse logistic; Mixed integer linear programming.

I. INTRODUCTION

Today, industrial organizations are working in a business environment characterized by a high competitiveness and short product life cycle. To meet different segments of customer's demands, companies must innovate and diversify their products by introducing constantly new products to the market.

New product design implies, certainly, the configuration of its corresponding supply chain. A better performance is achieved only if the designed product meets customer's needs on one hand and optimizes supply chain functions on the other hand.

Also, it have been pointed out that there exists a close connection between product design and supply chain performance. Novak and Eppinger [1] have explored by an empirical study, the link between product architectures and vertical integration decisions of supply chains. Their analysis showed that the companies optimizing the requirements of their product architectures as well as the capacities of their supply chains will outperform the firms focusing only on supply chain structures or product characteristics.

Research studies showed that 85% of logistics costs are driven by design choices [2] and over 70% of product cost is determined by decisions during this phase of development [3].Also, most benefits of collaboration among supply chain partners lie in the design phase of the product lifecycle since the cost of design changes increases as the design phase of the product lifecycle ends and the manufacturing phase starts [4]. Therefore, it is important to integrate product architecture decisions and supply chain decisions during the early stages of the product development [5, 6].

Growing awareness about raw materials shortage and environmental problems conduct enterprises to rethink their strategies to ensure the sustainability of their operations. Environmental criteria must be taken into account as well as economic feasibility. Thus, used-product collection and recovery activities are integrated to their regular supply chain. We talk then about reverse supply chain [7, 8].

This work integrates forward and reverse network activities to configure a close loop supply chain. Moreover, supply chain design and product design are considered simultaneously. In fact, supply chain constraints are integrated into product design phase on one hand and product specificities are also considered while determining supply chain structure. We consider that we are in the case of

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redesigning an existing product .The design change affects either the components or the process of the product or both of them at once. Design team, proposes several alternatives to redesign the product. The aim of this study is to select from different alternatives the best redesign accompanied with its optimal close loop supply chain. For this purpose a mixed integer linear programming model is proposed.

This paper is organized as follows: In Section 2, a literature review on product and supply chain design is presented. Section 3 is dedicated to reverse logistics concept. Section 4 presents the problem. The optimization mathematical model is given in section 5. Before the conclusion, a numerical example is presented in section 6 to illustrate the application of the model proposed.

II. PRODUCT-SUPPLY CHAIN DESIGN

In the literature, the most studied problems of designing a product and its supply chain follow crossword approaches [9].

A first approach consists on integrating product design constraints in supply chain design by taking into account the bill of material (BOM) of the product (assembly constraints).Works following this approach assume that product’s bill of material is well defined and already known ,so product design is finalized.

Cordeau et al [10] and Paquet et al [11] have proposed mixed linear programming models to design a multi-echelon supply chain and multi-product, considering assembly constraints and detailed bills of materials.

A second cross approach consists on integrating logistic constraints in the product design. The literature suggests some methods facilitating the integration between engineering and logistics actors like Design For Logistics (DFL) [12] and Design for Supply Chain Management (DSCM) [13].These methods define rules to optimize logistics costs by taking into account the logistical constraints in product design. Works on DFL and DFSCM promote the use of concepts such as modular design, delayed differentiation and components standardization to lower costs related to diversity management, storage and transportation of products[14],[15].In the same context, Nishigushi[16], Handfield and Nichols [17] have put their interest on supplier integration at the early phases of product design. Nevertheless, these works on logistical constraints integration assume that the supply chain already exists.

A third recent approach considers the design of a product and its related supply chain simultaneously. Supply chain design must be in interaction with product design process. On one hand, supply chain constraints must be integrated into product design phase. On the other hand, product specificities should be considered while determining supply chain structure. Therefore, the supply chain must be flexible and responsive to eventual product redesigns.

Works on simultaneous design of a product and its supply chain are very recent. Baud-Lavigne et al used a mathematical model in mixed linear programming to optimize the supply chain simultaneously with products standardization [18]. They illustrated impacts of product or component standardization on supply chain structure. El hadj khalaf et al have proposed a model to choose simultaneously modules to be produced and their suppliers, under final assembly time constraint [19]. El Maraghy et Mahmoudi [20] have proposed a multi-period model that simultaneously optimizes the supply chain and product nomenclature. They defined several alternative BOMs, one being selected in the optimal solution. This approach needs a complete enumeration of all product configurations. Labbi et al [21] have proposed a dynamic model that optimizes simultaneously the redesign of a product and its upstream supply chain considering the product lifecycle.

The simultaneous optimization of product and supply chain design is a difficult problem. Due to the complexity of the induced models, very few models address the integrated problems. They use a very large number of variables to model the problem in a comprehensive manner and are therefore difficult to solve.

III. REVERSE LOGISTICS AND CLOSE LOOP SUPPLY CHAIN

With the awareness about natural resources scarcity, companies start to adopt the green thinking by integrating the reverse logistics operations in their regular chain. The design of the green supply chain include design decisions of the supply chain integrating closed-loop reverse logistics, decisions regarding the reduction of carbon emissions, capacity utilization, energy and materials resources and regulatory factors concerns [22].Regarding the product lifecycle, the structure of a green supply chain is cyclic. Materials and information flow are exchanged in both directions between actors. This sets up a value-loop, which integrates all stages of the product lifecycle [23].

Research studies dealing with reverse logistics chain and closed-loop supply chain network design have known a significant increase. Neto et al [24] introduced a static model of linear programming in mixed integer numbers for the design of integrated closed-loop supply chain. The model allows trade-offs assessment between cost reduction and reducing the total environmental impact of the life cycle, under the regulatory constraints. Amin and Zhang [25] propose a stochastic model based on linear mixed integer programming. The model is bi-objectives; it minimizes the total cost of the integrated supply chain and maximizes use level of recyclable materials and clean technologies. Chaabane et al [26] propose a linear program in mixed integer numbers for the design of a green supply chain. The model is multi-period, multi-product and bi-objectives. It is formulated based on the principles of the life cycle analysis.

Amin and Zhang [27] developed an optimization model for closed loop supply chain configuration and supplier selection, simultaneously. Lundin [28] examined the effects of design changes of a closed loop supply chain network by a mathematical model. In this work, we develop an optimization model for the simultaneous design of a product and its supply chain integrating reverse logistics. We study the case of a product redesign.

IV. PROBLEM DESCRIPTION

Fig. 1 shows the structure of the close loop supply chain considered. The network consists of components' suppliers, subcontractors, a production plant, distribution centers, demand markets and recycling centers. It is supposed that the production plant is able to manufacture new products and remanufacture returned products. In the forward channel, the supply chain consists of suppliers, subcontractors, production plant, distribution centers and clients. In the reverse channel, products at the end of their life cycle are recovered from the end customers and carried to recycling centers .The returns are then carried to the direct chain to form a closed loop supply chain. It is assumed that products coming out recycling centers could be either final products or components.

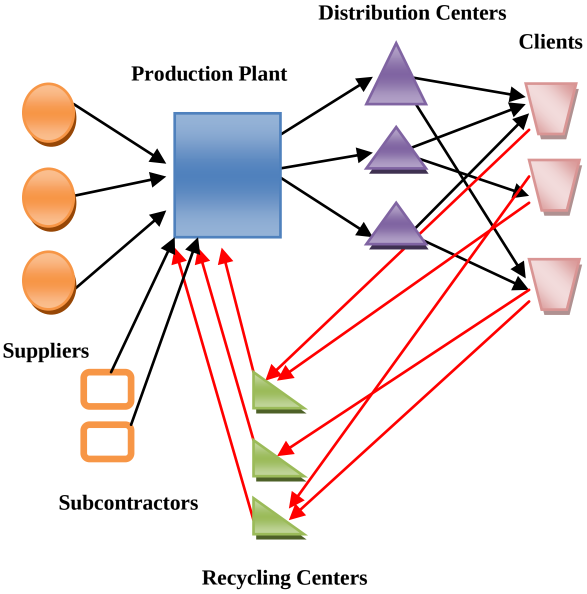

Fig 1. Structure of closed supply chain network adopted

We are in the case of redesign of a product and we want to determine optimally the associated closed loop supply chain. At the product design phase, several design alternatives are proposed by the design team. To each design alternative corresponds a supply chain configuration. The design of the product and its supply chain that will be chosen is that which provides the best cost and incorporates as many partners of the initial supply chain.

We consider that the product P is defined by its components Ci. Fig. 2 shows the adopted definition of product's bill of materials.

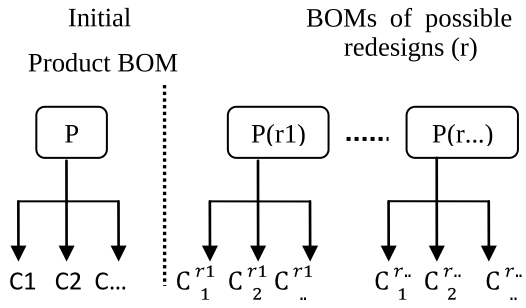

Fig.2. Product nomenclature definition.

The redesign of this product may include the following three cases:

- Redesign nomenclature consists of only components which are common with the initial product nomenclature having similar or different nomenclature's coefficients. This implies that product's design change affects the manufacturing process.
- The redesigned product is composed of new components that have never existed in the old nomenclature.
- The redesigned product combines common components and new ones.

To assemble this product according to each redesign alternative, we could either keep the same existing assembling technology (machine) or implement new technologies or have a mixture between old and new technologies.

To determine supply chain configuration, we propose a mixed integer linear programming (MILP) model. Section 5 shows in details the MILP building.

V. MATHEMATICAL MODEL

A. Mathematical model formulation

1) Problem assumptions

The considered assumptions are as follows:

- The demand for components is known.
- Suppliers undertake the transportation of required components.
- Components transportation unit cost is in the range of minimum and maximum capacity of each supplier.
- Each component can have a different quality index, that's why a segmentation based on the desired quality index is done earlier. In other words, all suppliers selected for a

component must deliver it with the same quality index to have homogeneous quantities.

- The producing company may call upon subcontractors to meet the product demand.
- The producing company pays a fixed cost including contract and partnership costs for suppliers and subcontractors newly introduced. This is for favoring suppliers and subcontractors that already exist at the starting supply chain.
- Each subcontractor must respect the product's quality level required by the producing company.
- Each subcontractor must respect the product's quality level required by the producing company.
- Only one mode of transportation is considered.
- Distribution centers assure the storage, the treatment and the transport of finished products.
- All the customers are served through distribution centers and not directly from the plants.
- New distribution centers could be introduced in case of the lack of capacity of the existing ones or when the new product design requires special conditions of storage that existing warehouses couldn't assure.
- The production plant pays a contracting cost to distributers and recycling centers newly introduced.
- Recycling centers could return either component or final products to the production plant.
- Recycling centers assure the transport of the returns.
- All assembling and recycling technologies (machines) added or implanted must not exceed a maximum number in order to respect the plant capacity and the investment budget for this new product.

2) Problem sets and data

To gather the three possible redesigns cited above into a single definition of redesign components set, we propose the following definition:

- C^r : set of redesign components, indexed by c, such as:

C^r= ($\mathrm{C}^0 \cap C^r$) U ($C^r \setminus \mathrm{C}^0$) with C^0: the set of components existing in the starting nomenclature.

Similarly,

- F^r : The set of redesign suppliers, indexed by f, such as:

F^r= ($\mathrm{F}^0 \cap F^r$) U ($F^r \setminus \mathrm{F}^0$) with F^0: the set of existing product's suppliers.

- M^r : The set of machines required for the redesign (r), indexed by m, such as:

M^r= ($\mathrm{M}^0 \cap M^r$) U ($M^r \setminus \mathrm{M}^0$) with M^0: the set of machines used to produce the existing product.

- N^r : The set of machines required for recycling the redesign (r), indexed by n, such as:

N^r= ($\mathrm{N}^0 \cap N^r$) U ($N^r \setminus \mathrm{N}^0$) with N^0: the set of machines necessary for recycling the existing product.

- S^r: the set of product subcontractors, indexed by s, such as:

S^r= ($\mathrm{S}^0 \cap S^r$) U ($S^r \setminus \mathrm{S}^0$) with S^0: the set of existing product's subcontractors.

- D^r : set of distribution centers corresponding to the alternative (r),indexed by d, such as :

D^r= ($\mathrm{D}^0 \cap D^r$) U ($D^r \setminus \mathrm{D}^0$) with D^0 the set of distribution centers employed to distribute the existing product.

- K^r : set of clients corresponding to the alternative (r),indexed by k, such as :

K^r= ($\mathrm{K}^0 \cap K^r$) U ($K^r \setminus \mathrm{K}^0$) with K^0 the set of clients of the existing product.

- G^r : set of recycling centers corresponding to the alternative (r),indexed by g, such as :

G^r= ($\mathrm{G}^0 \cap G^r$) U ($G^r \setminus \mathrm{G}^0$) with G^0 the set of clients of the existing product.

- T: the decision horizon indexed by t.
- R: the set of possible redesigns indexed by r.

The model includes the following data:

$\boldsymbol{CA_{c,f,t}}$: Purchasing unit cost of supplier **f** for the component **c** in period **t**.

$\boldsymbol{CT_{c,f,t}}$: Transport unit cost of supplier **f** for the component **c** in period **t**.

$\boldsymbol{CS_{c}}$ **:** Storage unit cost of the component **c.**

$\boldsymbol{CF_{f,t}}$: Partnership and collaboration fixed cost of the supplier **f** in period **t.**

$\mathbf{D_{c,t}}$: Demand for component **c** in period **t.**

$\mathbf{Capmax_{f,c,t}}$: Maximum capacity of the supplier **f** to deliver the component **c** in period **t.**

$\mathbf{CapTmax_{f,c,t}}$: Transport maximum capacity of the supplier **f** for the component **c** in period **t**.

$\mathbf{CapTmin_{f,c,t}}$: Transport minimum capacity of the supplier **f** for the component **c** in period **t**.

$\mathbf{Chp_{m,t}}$: Production hourly cost for a unit of the finished product in the machine **m** in period **t**.

$\mathbf{CFA_{m,t}}$: fixed cost of machine **m** implementation in period **t**.

$CTD_{d,t}$: Transport unit cost of the finished product from plant to a distributor **d** in period **t**.

$\mathbf{CFS_{m,t}}$: Removal or jobless cost of the machine **m** in period **t**.

$Chrf^{\,g}_{n,t}$: Recycling hourly cost for a unit of the finished product in the machine **n** in period **t**.

$Chrc^{\,g}_{n,t}$: Recycling hourly cost for a unit of the component **c** in the machine **n** in period **t**.

$\mathbf{CFRA_{n,t}}$: fixed cost of recycling technology **n** implementation in period **t**.

$\mathbf{CFRS_{n,t}}$: Removal or jobless cost of recycling technology **n** in period **t**.

$\mathbf{CF_s}$: Fixed cost related to selection of subcontractor s in period t.

$CST_{s,t}$: outsourcing cost of the subcontractor **s** for the finished product in period **t**.

$\mathbf{D_{k,t}}$ **:** Demand of the client **k** for the final product in period **t**.

$Capmax\_prod_{m,t}$: Production maximum capacity of the machine **m** in period **t**.

tpu_m : Production unit time of the finished product in a machine **m** in period **t**.

$\mathbf{NEI^0{}_m}$: Number of copies of a machine **m** originally existing at the producing company.

$\mathbf{MaxNE_{m,t}}$: Maximum number allowed for the addition of a machine **m** in period **t**.

$Capmax\_st_{s,t}$: Production maximum capacity of the subcontractor **s** in period **t**.

$MaxST_t$: Maximum quantity allowed for outsourcing the finished product in period t.

$Capmax\_recy_{n,t}$: Recycling maximum capacity of the machine **n** in period t.

$\mathbf{CF_g}$: contracting cost related to selection of recycling center **g** in period t.

$tpuf_n$: Recycling unit time of the finished product in a machine **n** in period **t**.

$tpuc_n$: Recycling unit time of the component c in a machine **n** in period **t**.

$\mathbf{NEI^0{}_n}$: Number of copies of a machine **n** originally existing at the recycling plant.

$\mathbf{MaxNE_{n,t}}$: Maximum number allowed for the addition of a machine **n** at the recycling plant in period **t**.

$\mathbf{CTGc_{g,t}}$: Transportation cost of the recycled component from recycling center **g** to the production plant in period **t.**

$\mathbf{CTGf_{g,t}}$: Transportation cost of the recycled finished product from recycling center **g** to the production plant in period **t.**

$\mathbf{CTKG_{k,g,t}}$: Transportation cost of the returned product from client **k** to recycling center **g** in period **t.**

r_c : Maximum ratio allowed for recovering component c.

r_f : Maximum ratio allowed for recovering finished product.

$\mathbf{CSD_{d,t}}$: Storage unit cost of the distribution center **d** in period **t**.

$\mathbf{CTK_{d,k,t}}$: Transportation cost of the finished product from distribution center **d** to client **k** in period **t.**

$\mathbf{CF_d}$: Fixed cost related to selection of distributor **d** in period t.

v_p : Volume occupied by the finished product **p**.

$\mathbf{V_d}$: Volume capacity reserved to the finished product **p** at the distribution center **d**.

3) Decision variables

$\mathbf{Q_{c,f,t}}$: Quantity of component c ordered from supplier f in period t.

$\mathbf{I_{c,t}}$: Inventory of component c at the end of period t.

$\mathbf{QP_t}$: Produced quantity of the finished product in period t**.**

$\mathbf{QS_{s,t}}$: Outsourced quantity of the finished product from the subcontractor **s** in period **t**.

$\mathbf{NEA_{m,t}}$: Number of copies added of a machine **m** in period **t**.

$NES_{m,t}$: Number of copies removed of a machine **m** in period **t**.

$QTD_{d,t}$: Quantity of the final product transferred from the production plant to distributer **d** in period **t.**

$QTK_{d,k,t}$: Quantity of the final product transferred from the distribution center **d** to client **k** in period **t.**

$NSD_{d,t}$: Quantity of the final product held in distribution center **d** in period **t.**

$QT_{k,g,t}$: Quantity of the returned product t from the client **k** to recycling center **g**in period **t.**

$QTGc_{g,t}$: Quantity of the returned component **c** transferred from the recycling center **g** to the production plant in period **t**

$QTGf_{g,t}$: Quantity of the returned finished product transferred from the recycling center **g** to the production plant in period **t.**

$NEAG^{\,g}_{n,t}$: Number of copies added of a recycling machine **n** at recycling center **g** in period **t**.

$NESG_{n,t}^{g}$: Number of copies removed of a recycling machine **n** at recycling center **g** in period **t**.

$S_{s,t}$**:** Binary variable for the allocation of subcontractors with $S_{s,t} = 1$, if the subcontractor **s** supplies the plant, and $S_{s,t} = 0$ otherwise.

$Z_{c,f,t}$: Binary variable for the allocation of components suppliers with $Z_{c,f,t} = 1$, if the supplier **f** is selected and $Z_{c,f,t} = 0$ otherwise.

$D_{d,t}$: Binary variable for the allocation of distribution centers **d** with $D_{d,t} = 1$,if the distribution center d is selected , and $D_{d,t} = 0$ otherwise.

$G_{G,t}$: Binary variable for the allocation of recycling centers **g** with $G_{G,t} = 1$, if the recycling center **g** is selected, and $G_{G,t} = 0$ otherwise.

4) Objective function

For each possible redesign, the objective function is to minimize supply, transportation, recycling and storage costs of components and production, outsourcing, adding or removing production and recycling machines costs and recycling and transportation of returned products. Our objective function is as follows:

$$
\begin{aligned}
&\text{Min} \quad \text{Min} \sum_{t\in T} \sum_{c\in C^r} \sum_{f\in F^r} (CA_{c,f,t} + CT_{c,f,t}) * Q_{c,f,t} \\
&r\in R \\
&+ \sum_{t\in T} \sum_{c\in C^r} CS_c * I_{c,t} \\
&+ \sum_{t\in T} \sum_{c\in C^r} \sum_{f\in F^r} CF_{ft} * Z_{c,f,t} \\
&+ \sum_{t\in T} \sum_{m\in M^r} (Chp_{m,t} * QP_t + CFA_{m,t} * NEA_{m,t} \\
&+ CFS_{m,t} * NES_{m,t}) + \sum_{t\in T} \sum_{d\in D^r} CTD_d * QTD_{d,t} \\
&+ \sum_{t\in T} \sum_{s\in S} (CST_{s,t} * QS_{s,t} + CF_s * S_{s,t}) \\
&+ \sum_{t\in T} \sum_{d\in D^r} (CSD_{d,t} * NSD_{d,t} + CF_d * D_{d,t}) \\
&+ \sum_{t\in T} \sum_{d\in D^r} \mathrm{CTK}_{d,k,t} * QTK_{d,k,t} \\
&+ \sum_{t\in T} \sum_{k\in K^r} \sum_{g\in G^r} \mathrm{CTKG}_{k,g,t} * QT_{k,g,t} \\
&+ \sum_{t\in T} \sum_{g\in G^r} (\mathrm{CTGc}_{g,t} * QTGc_{g,t} + \mathrm{CTGf}_{g,t} * \\
&QTGf_{g,t} + CF_g * G_{g,t}) \\
&+ \sum_{t\in T} \sum_{n\in N^r} (Chrf_{n,t}^{g} * QTGf_{g,t} + Chrc_{n,t}^{g} * \\
&QTGc_{g,t} + \mathrm{CFRA}_{n,t} * NEAG_{n,t}^{g} + \mathrm{CFRS}_{n,t} * \\
&NESG_{n,t}^{g}) \qquad (1)
\end{aligned}
$$

5) Constraints

- Suppliers capacity

$$Q_{c,f,t} \leq capmax_{c,f,t} * Z_{c,f,t} \quad \forall c,t \qquad (2)$$

Purchased quantity of the component **c** is limited by the production capacity of its supplier **f** .This is valid for each component at any planning period. With this constraint, we can also check if $Z_{c,f,t} = 0$ (the supplier f is not selected for the component c) then $Q_{c,f,t} = 0$.

- Transport capacity

$$CapTmin_{f,c,t} \leq Q_{c,f,t} \quad \forall c,f,t \qquad (3)$$

$$Q_{c,f,t} \leq CapTmax_{f,c,t} \quad \forall c,f,t \qquad (4)$$

Constraints (3) and (4) show that the quantity purchased of component c is bounded by the maximum and minimum transport capacities related to the corresponding supplier f.

- Component demand satisfaction

$$I_{c,t} + \sum_{f\in F^r} Q_{c,f,t} + \sum_{g\in G^r} QTGc_{g,t} \geq \mathbf{D}_{c,t} \quad \forall c,t \qquad (5)$$

This constraint shows that the sum of amounts of a component received from all suppliers and those recovered from all recycling centers in a period and the inventory of the previous period must meet the forecasted demand for this component in this period.

- Inventory constraint

$$\mathbf{I}_{c,t} = \sum_{f\in F^r} \mathbf{Q}_{c,f,t} + \sum_{g\in G^r} QTGc_{g,t} + \mathbf{I}_{c,t-1} - \mathbf{D}_{c,t} \qquad (6)$$
$$\forall c,f,g,t$$

Constraint (6) shows that the inventory in the end of the period (t) is a function of the quantity in stock at the end of the previous period (t-1), and purchased and recovered quantities minus the expected demand for period (t) for each component.

- Finished product demand satisfaction

$$\sum_{s\in S} QS_{s,t} + \sum_{g\in G^r} QTGf_{g,t} + QP_t \geq \sum_{k\in K} \mathbf{D}_{k,t} \quad \forall\ t \qquad (7)$$

Constraint (7) ensures that quantities produced, recovered and outsourced of the finished product must meet its estimated demand for each period.

- Subcontractors capacity

$$QS_{s,t} \leq Capmax\_st_{s,t} * S_{s,t} \quad \forall s,t \qquad (8)$$

Outsourced quantity of the finished product is limited by the production capacity of its subcontractor for each period.

- Outsourcing limitation

$$\sum_{s\in S} QS_{s,t} \leq MaxST_t \quad \forall t \qquad (9)$$

The quantities outsourced of the finished product received from all subcontractors must not exceed the allowed quantity for the outsourcing in each period.

- Producing capacity

$$QP_t * tpu_m \leq Capmax\_prod_{m,t} *$$
$$(\mathbf{NEI}^{0}_{m} + NEA_{m,t} - NES_{m,t}) \quad \forall m,t \qquad (10)$$

Constraint (10) ensures that the quantities produced of finished product respect production capacities of all available machines in each period.

- Machine implementation limitation

$$\mathbf{NEI}^{0}_{m} + NEA_{m,t} - NES_{m,t} \leq MaxNE_{m,t} \qquad (11)$$

Constraint (11) shows that all assembling technologies including machines that originally existed and those newly implanted must not exceed a maximum number in order to respect the plant capacity and investment budget for each machine type and each period.

- Demand satisfaction for distribution centers

$$\sum_{d\in D^r} \boldsymbol{QTK_{d,k,t}} \geq \boldsymbol{D_{k,t}} \qquad (12)$$

Constraint (12) shows that quantities of finished product delivered from all distributer centers should meet the demand of the client **k**.

- Distribution center capacity constraint

$$\boldsymbol{v_p} * \sum_{t\in T} \boldsymbol{QTD_{d,t}} \leq \boldsymbol{V_d} * \boldsymbol{D_{d,t}} \forall\, d,t \qquad (13)$$

This constraint shows that quantities of finished product transferred from production plant to a distributor center **d** is limited by its capacity of storage dedicated to the product. With this constraint, we can also check if $\boldsymbol{D_{d,t}} = 0$ (the production plant doesn't select the distributer **d**) then $\boldsymbol{QTD_{d,t}} = 0$.

- Flow conservation at distribution centers

$$\boldsymbol{NSD_{d,t}} = \boldsymbol{NSD_{d,t-1}} + \boldsymbol{QTD_{d,t}} - \sum_{k\in K^r} \boldsymbol{QTK_{d,k,t}} \qquad (14)$$

This constraint is about flow conservation at distribution centers, they must receive enough finished product from production plant in order to meet all the demands.

- Recycling capacity

$$\boldsymbol{QTGc_{g,t}} * \boldsymbol{tpuc_m} + \boldsymbol{QTGf_{g,t}} * \boldsymbol{tpuf_m} \leq \boldsymbol{Capmax\_recy_{n,t}} * \boldsymbol{G_{G,t}}$$

$$(\mathbf{NEI}^0_{\mathbf{n}} + \boldsymbol{NEAG}^{\,g}_{n,t} - \boldsymbol{NESG}^{\,g}_{n,t}) \forall\, \mathrm{n,t} \qquad (15)$$

Constraint (15) ensures that the quantities recovered of components and finished products in selected recycling centers respect recycling capacities of all available machines in each period.

- Machine implementation limitation

$$\mathbf{NEI}^0_{\boldsymbol{n}} + \boldsymbol{NEAG}^{\,g}_{n,t} - \boldsymbol{NESG}^{\,g}_{n,t} \leq \boldsymbol{MaxNE_{n,t}} \qquad (16)$$

Constraint (16) shows that all recycling technologies including machines that originally existed and those newly implanted must not exceed a maximum number in order to respect the recycling center capacity and investment budget for each machine type and each period.

- Forward and reverse channel constraint

$$\sum_{k\in K^r} \sum_{g\in G} \boldsymbol{QT_{k,g,t}} \leq \boldsymbol{QP_t} + \sum_{s\in S} \boldsymbol{QS_{s,t}} \qquad (17)$$

Constraint (17) ensures that forward channel is greater than reverse one.

- Returned product constraints

$$\sum_{k\in K^r} \sum_{g\in G} \boldsymbol{QT_{k,g,t}} * \boldsymbol{r_c} \leq \boldsymbol{QTGc_{g,t}} \qquad (18)$$
$$\sum_{k\in K^r} \sum_{g\in G} \boldsymbol{QT_{k,g,t}} * \boldsymbol{r_f} \leq \boldsymbol{QTGf_{g,t}} \qquad (19)$$

Constraints (18) and (19) show that the maximum percentage of recycling is respected.

- Non negativity and binary constraints

$$\boldsymbol{Q_{c,f,t}} \geq 0\, ; \quad \boldsymbol{I_{c,t}} \geq 0\, ; \boldsymbol{QS_{s,t}} \geq 0\, ; \quad \boldsymbol{QP_t} \geq 0$$

$$\boldsymbol{QTD_{d,t}} \geq 0\, ; \boldsymbol{QTK_{d,k,t}} \geq 0 \quad ; \boldsymbol{NSD_{d,t}} \geq 0$$

$$\boldsymbol{QT_{k,g,t}} \geq 0\, ;\ \boldsymbol{QTGc_{g,t}} \geq 0\ ;\ \boldsymbol{QTGf_{g,t}} \geq 0$$

$$\boldsymbol{Z_{c,f,t}}, \boldsymbol{S_{s,t}}, \boldsymbol{D_{d,t}}, \boldsymbol{G_{G,t}} \in \{0, 1\}$$

$\boldsymbol{NEA_{m,t}}, \boldsymbol{NES_{m,t}}, \boldsymbol{NEAG}^{\,g}_{n,t}, \boldsymbol{NESG}^{\,g}_{n,t}$: integers.

VI. NUMERICAL EXAMPLE

To have a better insight of the model, a numerical example is presented in this part. As this model aims to optimize product and supply chain design at a strategic (long term) level, time periods are considered to be product life cycle phases. In this example we considered four periods corresponding to introduction, growth, maturity and decline phases. We consider that a producing company wants to redesign a product P. The design team proposes three alternatives P1, P2, P3 for the redesign of the initial product P. In the three redesigns, there was either a change in the components or in the process or both of them at once. Fig 3 shows the nomenclature of initial product and that of the three alternatives proposed. We suppose that the initial product is composed by three components C1, C2, and C3. Components C4 C5, C6, and C7 are new ones.

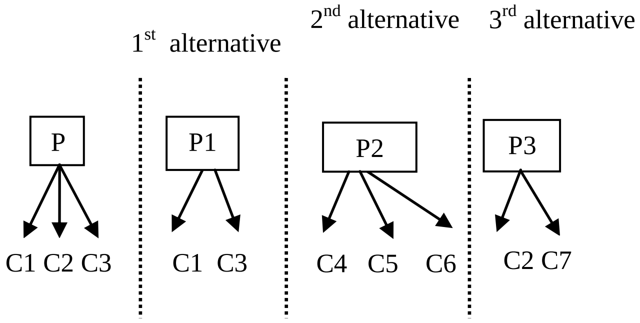

Fig.3.Nomenclature of the three proposed alternatives

The set of suppliers for each component are as follows:

C1= {F1, F2, F3}; C2= {F2, F3}; C3= {F1, F2};

C4= {F1, F4}; C5= {F2, F5}; C6= {F4, F5};

C7= {F3, F4};

The suppliers F4 and F5 are new suppliers to be introduced for new components. It's assumed that the purchasing cost, the transport cost and the defect ratio for each supplier to each component are the same in each period. To assemble the redesigned product according to each alternative, we will need either the existing machines used to assemble the initial product or new ones or both of them at once. Fig 4 shows the necessary machines for producing the redesigned product according to each possible redesign. We supposed that the initial product needs three machines M1, M2, and M3 to be assembled. M4 and M5 are new machines to be implanted.

We assume that there are four subcontractors (S1, S2, S3 and S4) for outsourcing the finished Products .S4 is a new subcontractor. It is also assumed that the maximum capacity of each subcontractor at each period is the same .Regarding assembling machines; we assume that the production unit cost, the production capacity, the implementation and the removal costs are the same for each period. The rest of the chain comprises three clients (K1, K2, K3), two distribution centers (D1, D2), two recycling centers (G1, G2) with two types of recycling machines(N1,N2).To achieve the design of the proposed alternatives, we will need to add a new distributor D3,a new recycling center G3 and new recycling machine N3.

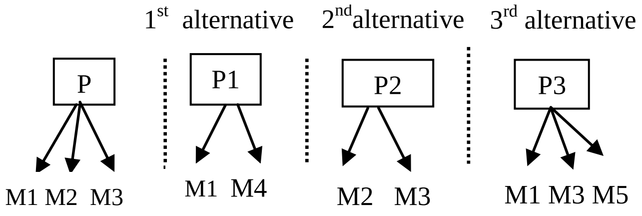

Fig 5. Production necessary machines

Tables (1) to (14) in the appendix show data have been used for the numerical example. The mathematical model was programmed and solved using the IBM Ilog Cplex 12.2. The program was solved for each of the three alternatives. Table 15 illustrates the results obtained.

TABLE 15. RESULTS TABLE.

	MILP Obj.	Sol.time (s)	variables	constraint	Coef.
P1	6.1848 e+004	810.71	266	734	4206
P2	6.9082 e+004	931.54	274	742	4222
P3	5.5937 e+004	921.39	319	855	5855

Comparing the MILP objective of the three alternatives, we conclude that alternative three (P3) is the optimal one. The values of decision variables of P3 are illustrated in table 11 of the appendix.

Results show that F2, F3 and F4 are the components' suppliers that have been selected for all periods. Regarding outsourcing, S2 is the only subcontractor to be selected in introduction phase. At maturity phase, S3 and S4 were added to S2 to respond to the high product demand. In period 4, the company will not need to outsource its product since product demand decreases in decline phase. To assemble P3, the company needs to implant two new machines (type M5) at the introduction phase. To respond to the growing demand of maturity phase, one machine (type M1), one machine (type M3) and also three machines (type M5) must be added. Results show also that the producing company receives recycled components and finished products from the three recycling centers in all periods. These recycled products added to quantities produced and outsourced cover the demand of all the three clients. Regarding distribution, the three distribution centers were used for string and distributing final products.

This numerical example shows how the supply chain configuration could change when we are redesigning a product. Also, it shows the dynamic aspect of the model since the supply chain related to redesigned product evolves in each product lifecycle phase. The resolution of the model showed the great number of variables, constraints and coefficients that have been handled which reflects the heaviness of the model even if it was simulated only for a product with very few components. Therefore the resolution of the model in a real industry case will be certainly very hard.

VII. CONCLUSION

This work contributes to a recent field which is the simultaneous design of the product and its supply chain. Moreover, it integrates also reverse logistic operations in order to treat all upstream and downstream functions of the supply chain from the product design till the recycling process. Logistics constraints are integrated in the product design phase on one hand, and the product attributes are considered when deciding about supply chain partners on the other hand.

We consider that we are in the case of designing a new product starting from an existent old one and we want to determine optimally its adequate closed loop supply chain. The supply chain configuration is designed from the existing partners of the existent supply chain related to the old product and by introducing new ones necessary for the new product achievement. At product design phase, several design alternatives are proposed. Each alternative corresponds to a closed loop supply chain. The aim is to optimally choose the best suppliers, subcontractors, distribution and recycling centers. For this purpose, a mixed integer linear programming (MILP) formulation was proposed.

The MILP is solved for each design alternative. The design of the product and its supply chain that will be chosen is that which provides the best cost and incorporates as many partners of the initial supply chain.

One of the limitations of this work is that we studied only the case of designing one product and its supply chain. In our future work, we will discuss the case of designing a family of product .Also the model contains many variables and constraints which makes it difficult to solve in terms of complexity especially in the case of industry-wide problems.

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APPENDIX . NUMERICAL EXAMPLE DATA

TABLE 1 .SUPPLIER'S QUANTITATIVE DATA

	$CA_{c,f,t}$					$CF_{f,t}$					$\mathrm{CT}_{c,f,t}$				
	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5
C1	6	7	6	-	-	0	0	0	-	-	6	5	6	-	-
C2	-	8	7	-	-	-	0	0	-	-	-	5	6	-	-
C3	6	5	-	-	-	0	0	-	-	-	6	4	-	-	-
C4	8	-	-	7	-	0	-	-	80	-	6	-	-	7	-
C5	-	8	-	-	9	-	0	-	-	80	-	7	-	-	8
C6	-	-	-	8	7	-	-	-	80	80	-	-	-	5	6
C7	-	-	9	8	-	-	-	0	80	-	-	-	6	6	-

TABLE 2 .MAXIMUM CAPACITY FOR EACH SUPPLIER

	Period 1					Period 2					Period 3					Period 4				
	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5
C2	-	70	60	-	-	-	50	90	-	-	-	80	100	-	-	-	70	60	-	-
C3	70	60	-	-	-	50	90	-	-	-	80	100	-	-	-	70	60	-	-	-
C4	40	-	-	60	-	60	-	-	90	-	70	-	-	120	-	40	-	-	90	-
C5	-	30	-	-	70	-	50	-	-	90	-	70	-	-	110	-	40	-	-	90
C6	-	-	-	80	80	-	-	-	90	100	-	-	-	80	120	-	-	-	70	90
C7	-	-	50	80	-	-	-	60	90	-	-	-	60	120	-	-	-	50	80	-

TABLE 3. MACHINES QUANTITATIVE DATA.

	$\mathbf{Chp}_{m,t}$					$\mathbf{CFA}_{m,t}$					$\mathbf{CFS}_{m,t}$					$Capmax\_prod_{m,t}$				
	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5
P1	1	-	-	2	-	100	-	-	130	-	50	-	-	60	-	40	-	-	50	-
P2	-	3	1	-	-	-	90	120	-	-	-	40	50	-	-	-	50	60	-	-
P3	1	-	1	-	2	100	-	120	-	110	50	-	50	-	40	40	-	60	-	50

TABLE 4. MACHINES QUANTITATIVE DATA

	tpu_m					$\mathbf{NEI}^0_m$				
	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5
P1	1	-	-	2	-	5	-	-	0	-
P2	-	2	1	-	-	-	2	1	-	-
P3	2	-	1	-	1	5	-	1	-	0

TABLE 5. FINISHED PRODUCT INFORMATION

	$\mathbf{D}_t$				$MaxST_t$			
	t1	t2	t3	t4	t1	t2	t3	t4
P1	80	130	160	100	20	40	40	20
P2	80	130	160	100	20	40	40	20
P3	80	130	160	100	20	40	40	20

TABLE 6. DEMAND FOR COMPONENTS

	$D_{c,t}$			
	t1	t2	t3	t4
C1	160	260	320	200
C2	80	130	160	100
C3	80	130	160	100
C4	80	130	160	100
C5	80	130	160	100
C6	80	130	160	100
C7	80	130	160	100

TABLE 7. DISTRIBUTOR VOLUME

V_d	D1	D2	D3
P1	900	1100	1000
P2	800	1000	900
P3	1000	1200	1100

TABLE 8. TRANSPORTATION COST

$\mathbf{CTKG_{k,g}}$	G1	G2	G3
K1	4	5	4
K2	6	5	6
K3	7	5	6

TABLE 9. TRANSPORTATION COST

$\mathbf{CTGf_g}$	G1	G2	G3
	4	5	4

TABLE 10. RECYCLING MACHINE DATA

$tpuf_n$	N1	N2	N3
P1	2	1	2
P2	2	3	2
P3	1	2	1

TABLE 11.MAXIMUM OF MACHINES IN EACH PERIOD

P1				P2				P3			
t1	t2	t3	t4	t1	t2	t3	t4	t1	t2	t3	t4
10	11	12	10	-	-	-	-	10	11	12	9
-	-	-	-	8	10	11	8	-	-	-	-
-	-	-	-	6	7	8	6	6	7	8	6
8	9	10	8	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	6	7	8	6

TABLE 12.DISTRIBUTOR DATA

	$\mathbf{CSD_{d,t}}$		
	D1	D2	D3
P1	5	6	5
P2	6	7	6
P3	5	6	6

TABLE 13. SUBCONTRACTOR QUANTITATIVE DATA

	$CST_{s,t}$				$\mathbf{CF_s}$				$Capmax\_st_{s,t}$			
	S1	S2	S3	S4	S1	S2	S3	S4	S1	S2	S3	S4
P1	6	7	8	-	0	0	0	-	30	40	30	-
P2	6	7	-	6	0	0	-	30	20	20	-	40
P3	-	7	8	6	-	0	0	40	-	20	10	40

TABLE 14. DEMAND OF THE FINAL PRODUCT

$D_{k,t}$	K1	K2	K3
t1	30	20	30
t2	50	40	40
t3	40	50	70
t4	30	40	30

Table16 .Decision variables values for redesign P3

	$\mathbf{Q_{c,f,t}}$	$\mathbf{QP_t}$	$\mathbf{QS_{s,t}}$	$\mathbf{NEA_{m,t}}$	$Z_{c,f,t}$	$\mathbf{I_{c,t}}$	$QTGc_{g,t}$
Introduction phase	$Q_{2,2,1} = 70$;$Q_{2,3,1} = 15$ $Q_{7,3,1} = 50$;$Q_{7,4,t} = 32$	QP_1 =60	$QS_{2,1}$=10; $QS_{3,1}$ =0 $QS_{4,1}$=0	$NEA_{1,1}$ =0 $NEA_{3,1}$ =0 $NEA_{5,1}$ =2	$Z_{2,2,t} = 1$ $Z_{2,3,t} = 1$ $Z_{7,3,t} = 1$;	$I_{2,1}= 24$ $I_{7,1} = 1$	$QTGc_{1,1}$ =6 $QTGc_{2,1}$ =3 $QTGc_{3,1}$ = 3

Growth phase	$Q_{2,2,2} = 50$; $Q_{2,3,2} = 75$ $Q_{7,3,2} = 60$; $Q_{7,4,2} = 72$	QP_2 =100	$QS_{2,2}$=14;$QS_{3,2}$ =6 $QS_{4,1}$=0	$NEA_{1,2}$ =1 $NEA_{3,2}$ =1 $NEA_{5,2}$ =3	$Z_{2,2,t} = 1$ $Z_{2,3,t} = 1$ $Z_{7,3,t} = 1$ $Z_{7,4,t} = 1$	$I_{2,2}$= 17 $I_{7,2}$ = 1	$\boldsymbol{QTGc_{1,2}}$ =12 $\boldsymbol{QTGc_{2,2}}$ =5 $\boldsymbol{QTGc_{3,2}}$ =8
Maturity phase	$Q_{2,2,3} = 80$; $Q_{2,3,3} = 100$ $Q_{7,3,3} = 60$; $Q_{7,4,4} = 120$	QP_3 =120	$QS_{2,3}$=10;$QS_{3,3}$ =10 $QS_{4,3}$=10	$NEA_{1,3}$ =1 $NEA_{3,3}$ =1 $NEA_{5,3}$ =3	$Z_{2,2,t} = 1$ $Z_{2,3,t} = 1$ $Z_{7,3,t} = 1$; $Z_{7,4,t} = 1$	$I_{2,3}$= 34 $I_{7,3}$= 18	$\boldsymbol{QTGc_{1,3}}$ =12 $\boldsymbol{QTGc_{2,3}}$ =10 $\boldsymbol{QTGc_{3,3}}$ =8
Decline phase	$Q_{2,2,4} = 68$;$Q_{2,3,4} = 16$ $Q_{7,3,4} = 40$; $Q_{7,4,4} = 80$	QP_4 =86	$QS_{2,4}$; $QS_{3,4}$; =0	$NEA_{1,4}$ =0 $NEA_{3,4}$=1 $NEA_{5,4}$ =2	$Z_{2,2,t} = 1$ $Z_{2,3,t} = 1$ $Z_{7,3,t} = 1$; $Z_{7,4,t} = 1$	$I_{2,4}$ = 17 $I_{7,4}$ = 6	$\boldsymbol{QTGc_{1,4}}$ =6 $\boldsymbol{QTGc_{2,4}}$ =3 $\boldsymbol{QTGc_{3,4}}$ =4

	$\mathbf{QTK_{d,k,t}}$	$\boldsymbol{S_{d,t}}$	$\mathbf{QTD_{d,t}}$	$\boldsymbol{D_{d,t}}$	$\boldsymbol{QTGf_{g,t}}$
Introduction phase	$QTK_{1,2,1} = 5$;$QTK_{2,1,1} = 5$ $QTK_{2,2,1} = 15$; $QTK_{3,1,1} = 25$;$QTK_{3,3,1} = 30$	$S_{2,1}$ =1	$QT_{1,1}$=5; $QT_{2,1}$=20 $QT_{3,1}$=55	$D_{2,1}$ =1 $D_{1,1}$ =1 $D_{3,1}$ =1	$\boldsymbol{QTGf_{1,1}}$ =3 $\boldsymbol{QTGf_{2,1}}$ =4 $\boldsymbol{QTGf_{3,1}}$ =3
Growth phase	$QTK_{1,1,2} = 50$;$QTK_{1,2,2} = 40$ $QTK_{1,3,2} = 40$;	$S_{2,2}$ =1 $S_{3,2}$ =1	$QT_{1,2}$=130;$QT_{2,2}$=0 $QT_{3,2}$=0	$D_{1,2}$ =1 $D_{2,2}$ =0 $D_{3,2}$ =0	$\boldsymbol{QTGf_{1,2}}$ =5 $\boldsymbol{QTGf_{2,2}}$ =3 $\boldsymbol{QTGf_{3,2}}$ =2
Maturity phase	$QTK_{1,3,3} = 70$;$QTK_{2,1,3} = 40$ $QTK_{2,2,3} = 50$;	$S_{2,3}$ =1 $S_{3,3}$ =1 $S_{4,3}$ =1	$QT_{1,3}$=70;$QT_{2,3}$=90 $QT_{3,3}$=0	$D_{1,3}$ =1 $D_{2,3}$ =1 $D_{3,3}$ =0	$\boldsymbol{QTGf_{1,3}}$ =5 $\boldsymbol{QTGf_{2,3}}$ =5 $\boldsymbol{QTGf_{3,3}}$ =4
Decline phase	$QTK_{1,2,4} = 2$; $QTK_{2,1,4} = 26$; $QTK_{3,1,4} = 2$ $QTK_{3,2,4} = 40$; $QTK_{3,3,4} = 30$	$S_{2,4}$; $S_{3,4}$; $S_{4,4}$ =0	$QT_{1,4} = 2$ $QT_{2,4} = 26$ $QT_{3,4}$ =72	$D_{1,4}$ =1 $D_{2,4}$ =1 $D_{3,4}$ =1	$\boldsymbol{QTGf_{1,4}}$ =5 $\boldsymbol{QTGf_{2,4}}$ =5 $\boldsymbol{QTGf_{3,4}}$ =5