

# Joint optimization for maintenance planning and quality deterioration

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**Abstract**—This paper investigates an integrated strategy joining non quality effects and preventive maintenance (PM) policy. We consider a single machine subject to random failure rate and producing a progressive deteriorating products. A preventive maintenance (PM) strategy with minimal repair is applied with non negligible durations of maintenance tasks. This study consists on developing an analytical model in order to determine an optimal integrated maintenance plan taking into account the quality deterioration of products. This strategy consist on selling products at a discount price due to the loss of quality caused by the machine degradation. The objective is to determine the optimal number of batches produced  $N^*$  before each preventive maintenance action which maximize the total profit (PT) per time unit for a full cycle  $T$ . A numerical example is presented in order to illustrate the proposed model. A sensitivity study is used to evaluate the influence of model parameters.

**Keywords:** Integrated maintenance strategy, Quality deterioration, Optimization.

## I. INTRODUCTION

The increased global competition calls farms to manage several functional areas successfully such as maintenance, quality, production and marketing. Consequently, managers and decision makers have to consider new management approaches integrating the interaction between parts or all of those functions. Quality and Maintenance are important aspects in any industrial process. They have been widely studied separately during decades. In fact, the earliest work on maintenance policies was elaborated by Barlow [4] and since that researchers has allocated considerable efforts on developing and optimizing preventive maintenance models which contributed to a good understanding of the properties and effectiveness of preventive maintenance policies under various conditions. During the last decade, practitioners and academicians recognized that there is a strong relationship between product quality and equipment maintenance. In fact, strategies dissociating maintenance and quality were imperfect and ineffective and it is commonly argued that preventive maintenance (PM) policies can contribute significantly in increasing equipment reliability as well as product quality. In addition, maintenance is considered among the three

main reasons of quality deterioration [15]. Then, an effective integration of these two components represent the favored means to avoid quality deficiencies. In this perspectives, the need for developing new integrated maintenance-quality strategies became evident and the simultaneous consideration of maintenance policies and quality deterioration has recently become an important research area.

Our objective in this study is to develop new analytical model joining maintenance and quality simultaneously for a manufacturing system composed of a single machine producing a single product. The machine is subject to progressive degradation according to an increased random failure rate which affects the product quality. We consider a PM strategy with minimal repair. The preventive and corrective actions have non-negligible durations. We aim to determine the optimal preventive maintenance plan taking into account maintenance costs and non-quality effects by determining the optimal number of batches produced before each preventive maintenance action.

The reminder of this paper is organized as follow: next section gives an overview of works that study maintenance policies and the integrated maintenance strategies. Section 3 introduced the problem statement and the notation used. In section 4, we present an analytical model. Section 5 is dedicated to present numerical example to illustrate the proposed model and results obtained and in the last section we present a sensitivity study to show the impact of the variation of some parameters on the optimal solution.

## Motivation

Practitioners and academicians recognize that the close link between product quality and equipment maintenance should no more go unnoticed. In fact, the integration of these two aspects is economically beneficial to organizations. Then, more research considering simultaneously maintenance policies and quality deterioration needs to be conducted to highlight this relationship. In this study, we present new mathematical model joining quality deterioration and maintenance policies in order to maximize the total benefit. The objective is to determine the optimal preventive maintenance plan taking into account maintenance cost and

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non quality effect.

## II. LITERATURE REVIEW

In this section, we present a state of the art of researchers which studied maintenance optimization models and integrated models.

Maintenance as a separate problem has widely discussed in the literature over the last decade. Works on maintenance policies have started with Barlow et al. [4] in 1960 and it was achieved with a huge number of contribution. M. Vasili et al. [18] present a brief review of existing maintenance optimization models: They discussed several reliable models and methods and investigated future prospects in this area. In Dekker [14] a review on applications of maintenance optimization is given.

Production, quality, and maintenance policy are important components of a manufacturing system. Companies' decision makers and researchers have realized that strategies dissociating these functional areas are no more efficient. The research community has allocated considerable efforts on modeling and optimizing integrated models in order to understanding and solving these problems. Some researchers have studied models joining maintenance and quality, others have studies the combination of maintenance and production and considerable works have interested in integrated models joining these three aspects.

M. Ben Daya [2] proposed an integrated model for the joint determination of economic production quantity and preventive maintenance level for imperfect production process. He proved that performing maintenance gives way to a reduction of quality control related cost.

M. Radhoui et al. [1] studied a joint preventive maintenance and quality control policy for a manufacturing system producing conforming and non-conforming products. The number of non-conforming items is the decision variable for the maintenance intervention. A buffer stock is constructed whenever the rate of non-conforming products attains a specific limit. The objective was to determine this limit and the size of buffer stock that minimize the overage total cost per unit time integrated maintenance quality and inventory cost.

A. Chelbi and N. Rezg [5] consider a policy based on the age of the equipment. They determine the optimal age of the system, the most appropriate time to be submitted to preventive maintenance, and the optimal size of the buffer stock. The aim is to minimize the total cost per time unit, taking into account a minimum required system availability level.

A.Ollila and M. Malmipuro [15] present in their study that maintenance has a major impact on product quality. In a case study, they showed that the maintenance is among the three most important causes of quality deterioration.

Control chart as a commonly applied tool of statistical process control has been extensively used to monitor and

even reduce process variation by identifying and eliminating sources of variation. Many models addressing statistical process control and maintenance simultaneously have appeared in the literature. The first model for the economic design of X bar control charts was introduced by J. A. Duncan [6]. Duncan's model stimulated most of the work in this area and has been extended by several authors. For most case, its not possible to separate statistical process control from preventive maintenance [11]. M. Ben Daya [12] developed an integrated model for the joint optimization of the economic production quantity, the economic design of x-control chart, and the optimal maintenance level, for a deteriorating process where the incontrol period follows a general probability distribution with increasing hazard rate. In addition, M. A. Rahim [13] and M. Ben-Daya and M. A. Rahim [7] proposed an integrated model based on X control chart and preventive maintenance in which the in-control time follows a probability distribution with increasing hazard rate. K. Linderman et al. [8] developed an integrated model, which applies an EWMA control chart, to reduce cost. They assumed perfect maintenance and expressed three scenarios for their model. In two scenarios, process continues until planned maintenance time. In the third scenario, control chart detects out-of-control condition.

## III. PROBLEM STATEMENT

### A. Description of the problem

We consider a single machine producing a single product type and subject to random failures. It is assumed that the product quality is subject to an increasing deterioration due to the machine degradation. Each batch produced loses quality compared to the previous and it is sold at a discount price. The relation between the price of two consecutive batches is :

$$P_i = \alpha_i \times P_{i-1}.$$

with  $\alpha_i$  is the quality degradation ratio of the  $i^{th}$  batch.

We note that :

$$P_1 = \alpha_1 \times P_{max}$$

$$P_2 = \alpha_2 \times P_1 = \alpha_2 \times \alpha_1 \times P_{max}$$

.

$$P_N = \alpha_N \times \alpha_{N-1} \times \dots \times \alpha_1 \times P_{max}$$

$$\text{Consequently } P_i = \prod_{j=1}^i \alpha_j \times P_{max}.$$

We assume that the machine failure rate have an increased failure rate following Weibull distribution. The choice of this distribution law is explained by the importance of the weibull distribution in characterizing equipment failure [17] and its applicability to a wide range of industrial settings[16]. The failure rate  $\lambda(t)$  is given by (1)

$$\lambda(t) = \frac{b}{\epsilon} \times \frac{t^{b-1}}{\epsilon} \quad (1)$$

With

- $b$ : Shape parameter
- $\epsilon$ : Scale parameter

We recall that we adopt an increasing failure rate and we assume that  $b \geq 2$ .

We applied a (PM) policy with minimal repair at failures with non-negligible durations of preventive and corrective maintenance tasks. Preventive maintenance actions are performed each  $N$  batches produced. Following each PM action, the manufacturing system is considered restored to a state as good as new and the product quality returns to 100%. The corrective maintenance action is undertaken only when a breakdown occurs.

Our strategy is illustrated in figure (1).

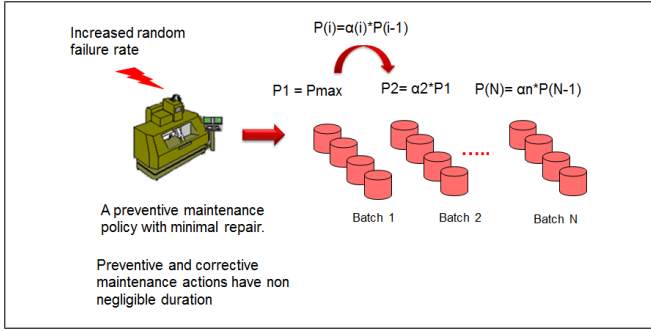


Fig. 1. Problem statement

## B. Objective

The objective of this study is to determine the optimal number of produced batches before the preventive maintenance action taking into account the preventive maintenance cost, the selling price and the quality deterioration degree of products. We aim to maximize the total profit per time unit for a full cycle.

## IV. ANALYTICAL STUDY

In this section, we formulate a mathematical model joining maintenance and quality. The aim is to find analytical model that enable us to determine the optimal value of the decision variable  $N^*$  maximizing total profit (PT) per time unit for a full cycle.

### A. Notation

In order to develop the analytical model related to our problem, we present the following notations :

$PT$  : Average total profit per time unit of a full cycle

$S_p$  : Total selling price

$CP$  : Total Production cost

$C_p$  : Unit production cost

$M_p$  : Preventive maintenance cost

$C_{cm}$  : Corrective maintenance cost

$M_c$  : Corrective maintenance action cost

$T$  : Average duration of a full cycle

$\lambda(t)$  : Failure rate

$\int_a^b \lambda(t)dt$  : Average number of failure per unit of time during  $[a, b]$

$N$  : Number of batches produced in a full cycle

$Proc$  : Average processing time

$\mu_p$  : Average duration of preventive maintenance action

$\mu_c$  : Average duration of corrective maintenance action

$P_{max}$  : The selling price of the first batch

$P_i$  : The selling price of the  $i^{th}$  batch

$\alpha_j$  : Degradation quality ratio of  $j^{th}$  batch  $\in [0, 1]$

$\alpha$  : Overage degradation of quality

### B. Mathematical model

The expected total profit per time unit (PT) for the first integrated model includes:

- Total selling price
- Corrective maintenance Cost
- Preventive maintenance cost
- Total production cost

The expected total profit by time unit for this integrated model is given by (2):

$$PT = \frac{S_p - (M_p + C_{cm} + CP)}{T} \quad (2)$$

With

$$S_p = \sum_{i=1}^N P_{max} \times \prod_{j=1}^i \alpha_j$$

$$C_{cm} = M_c \times \int_0^{N*Proc} \lambda(t)dt$$

$$CP = C_p \times N$$

$$T = N \times Proc + \mu_p + \mu_c \times \int_0^{N*Proc} \lambda(t)dt$$

The expected total profit per time unit for this model is

presented by the equation (2).

$$PT(N) = \frac{\sum_{i=1}^N P_{max} \times \prod_{j=1}^i \alpha_j}{N \times Proc + \mu_p + \mu_c \times \int_0^{N \times Proc} \lambda(t) dt} - \frac{M_p + M_c \times \int_0^{N \times Proc} \lambda(t) dt + C_p \times N}{N \times Proc + \mu_p + \mu_c \times \int_0^{N \times Proc} \lambda(t) dt} \quad (3)$$

We assume that we have a constant degradation of the product quality. We note  $\alpha$  the overage ratio of quality degradation. Then, the total profit (PT) can be written as :

$$PT(N) = \frac{\sum_{i=1}^N P_{max} \times \alpha^{i-1}}{N \times Proc + \mu_p + \mu_c \times \int_0^{N \times Proc} \lambda(t) dt} - \frac{M_p + M_c \times \int_0^{N \times Proc} \lambda(t) dt + C_{pu} \times N}{N \times Proc + \mu_p + \mu_c \times \int_0^{N \times Proc} \lambda(t) dt} \quad (4)$$

The optimal number of batches  $N^*$  will be obtained by maximizing (3).

**Lemma 1.** *There exist an optimal solution maximizing the total profit (PT) par time unit.*

*Proof.* We recall that  $b$  is the shape parameter of the failure rate  $\lambda(t)$ .

- For  $b = 2$

$$\lim_{N \rightarrow 0} PT(N) = \frac{-M_p}{\mu_p}$$

$$\lim_{N \rightarrow +\infty} PT(N) = \frac{-M_c}{\mu_c}$$

- For  $b > 2$

$$\lim_{N \rightarrow 0} PT(N) = \frac{-M_p}{\mu_p}$$

$$\lim_{N \rightarrow +\infty} PT(N) = -\infty$$

Hence, for all  $b \geq 2$ , there exists a finite  $N^* \in ]0, +\infty[$  which maximize  $PT(N)$ .  $\square$

## V. NUMERICAL EXAMPLE

A numerical example is presented in this section in order to illustrate the analytical model developed in the previous section. The input data for the numerical example are summarized in table I below ('mu' stand for monetary units and 'tu' for time unit).

We have used MATHEMATICA® software to obtain the optimal solution.

Solving the problems formulated by equation (3), we obtained optimal results presented in table II.

This table shows that the preventive maintenance action must be taken every 7 batches.

The figure 2 presents the variation of the total profit (PT) for a full cycle. It shows clearly the existence of a maximum which corresponds to  $N^*$ .

TABLE I  
NUMERICAL DATA FOR THE FIRST STRATEGY

System time to failure	F(t): Weibull distribution Shape parameter $\beta = 2$ Scale parameter $\epsilon = 100$
$P_{max}(mu)$	400
$M_p(mu)$	500
$M_c(mu)$	1200
Proc (tu)	3
$C_{pu}(mu)$	100
$\mu_p(tu)$	10
$\mu_c(tu)$	80
$\alpha$	0.92

TABLE II  
OPTIMAL RESULTS

$N^*$	PT*
7	27.74

We recall that the aim of our work is to found the optimal number of batches produced  $N^*$  before the preventive maintenance action which maximizes the total profit (PT) per time unit.

## VI. SENSITIVITY STUDY

In this section, we intend to study the sensitivity of the proposed model to the variation of some parameters including the selling price  $P_{max}$ , corrective maintenance cost  $M_c$  and the quality degradation ratio  $\alpha$ .

the variation of model's parameters are presented in following tables IV, V and VI. Stationary variables used in this sensitivity study are grouped in table III.

TABLE III  
STATIONARY VARIABLES

$M_p$	$C_{pu}$	$\mu_p$	$\mu_c$	Proc	$b$	$\epsilon$
500	100	10	80	3	2	100

TABLE IV  
RESULT OF THE VARIATION OF  $P_{max}$

$P_{max}$	$M_c$	$\alpha$	$N^*$	PT*
250	1200	0.92	8	3.93
450	1200	0.92	7	35.74

TABLE V  
RESULT OF THE VARIATION OF  $\alpha$

$\alpha$	$M_c$	$P_{max}$	$N^*$	PT*
0.92	1200	450	7	35.74
0.98	1200	450	10	53.24

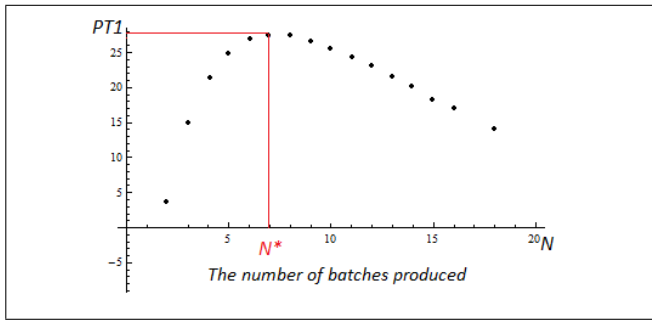


Fig. 2. The variation of total profit by time unit (PT) for a full cycle

TABLE VI  
RESULT OF THE VARIATION OF  $M_c$

$M_c$	$\alpha$	$P_{max}$	$N^*$	$PT^*$
1200	0.92	450	7	35.74
2000	0.92	450	6	34.3

Given the results shown in tables IV, V, VI, we note that when we increase the selling price, the number of batches produced before making a preventive maintenance decreases and the total expected profit for a full cycle by time unit increases. On the other hand, the increase of the corrective maintenance cost results the decrease of the number of batches produced before preventive intervention and the total profit. Moreover, when we increase the quality degradation ratio, the number of batches produced increases as well as the total profit. Table VII summarizes the influence of the variation of some parameters on the final results.

TABLE VII  
IMPACT OF THE VARIATION OF  $P_{max}$ ,  $M_c$  AND  $\alpha$

$P_{max}$	↗	$N^*$	↘	$PT^*$	↗
$M_c$	↗	$N^*$	↘	$PT^*$	↘
$\alpha$	↗	$N^*$	↗	$PT^*$	↗

## VII. CONCLUSION

The current paper presented an integrated maintenance quality problem applied to manufacturing system subject to progressive degradation according to Weibull distribution law of failure. We proposed an integrated optimization of the maintenance schedule and the quality deterioration in order to derive an optimal integrated maintenance-quality strategy taking into account the selling price, preventive maintenance cost and quality deterioration. Our approach consists on selling products at a discount price due to the loss of quality caused by the machine degradation and the objective is to determine the optimal number of batches produced before each preventive maintenance action. The contribution of this study is the planning of the preventive maintenance action after a number of batches produced contrarily to previous works in which the preventive maintenance action is related to the time. In

addition, the preventive and corrective maintenance actions have non negligible durations.

The proposed model has been illustrated with a numerical example. A sensitivity study is used to present the influence of the variation of some model parameters.

Extensions to this work are under consideration. One of them consists in the investigation of situations in which preventive maintenance actions are imperfect.

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